A Theory of Zombie Lending

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ABSTRACT

An entrepreneur borrows from a relationship bank or the market. The bank has a higher cost of capital but produces private information over time. While the entrepreneur accumulates reputation as the lending relationship continues, asymmetric information is also developed between the bank/entrepreneur and the market. In this setting, zombie lending is inevitable: once the entrepreneur becomes sufficiently reputable, the bank will roll over loans even after learning bad news, for the prospect of future market financing. Zombie lending is mitigated when the entrepreneur faces financial constraints. Finally, the bank stops producing information too early if information production is costly.

Keywords: private learning, experimentation, reputation, relationship banking, information monopoly, debt rollover, zombie lending, adverse selection, dynamic games

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Zombie firms – firms whose operating cash flows persistently fall below their interest payments – are common in the real world. According to a recent study by Banerjee and Hofmann (2018), zombie firms make up about 12% of all publicly traded firms across 14 advanced economies. These firms are detrimental to the real economy as they crowd out credit to their healthy competitors and thereby reduce aggregate productivity and investment. Indeed, zombie lending has long been perceived as the main reason behind Japan's "lost decade" in the 1990s (Caballero, Hoshi, and Kashyap (2008), Peek and Rosengren (2005)), and more recently, Acharya, Eisert, Eufinger, and Hirsch (2019) and Blattner, Farinha, and Rebelo (2019) show that Europe's economic recovery from the debt crisis has been plagued by bank lending to zombie firms. It is therefore natural to ask why banks extend loans to firms that are likely unable to repay their loan obligations.

One possible explanation is related to bank capital (e.g., Bruche and Llobet (2013)). In particular, by extending "evergreen" loans to their impaired borrowers, banks in distress gamble for resurrection, hoping that borrowing firms regain solvency or at least delay taking a balance sheet hit. However, as the Federal Deposit Insurance Corporation (FDIC) documents, well-capitalized banks also sometimes extend credit to distressed relationship borrowers.[1] These observartions raise the question of whether zombie lending is a natural and inevitable consequence in bank lending.[2]

In this paper, we build a dynamic model of relationship lending and argue that even absent concerns about bank capital, zombie lending is inevitable but self-limiting.[3] Our explanation hinges on the assumption that banks and private lenders have an information advantage over market-based lenders. A borrower's reputation therefore grows with the length of its lending relationship, because bad loans are initially liquidated. This reputation growth gives a bank incentives to roll over bad loans – evergreening – before passing the buck to the market. Zombie lending is therefore inevitable. However, if the bank consistently rolls over bad loans, it can destroy the reputation benefits acquired from the lending relationship as well as the bank's incentive to engage in zombie lending in the first place. As a result, projects found to be bad early on are liquidated, and thus no liquidation improves a borrower's reputation or perceived quality. In this sense, zombie lending is also self-limiting. The bank's liquidation policy early on offers incentives to

[1]For example, FDIC (2017, p.24) shows that First NBC Bank, a bank headquartered in New Orleans, Louisiana and failed in 2017, was considered *Well Capitalized* from 2006 through February 2015. "From 2008 through 2016, examiners criticized the bank's liberal lending practices to financially distressed borrowers, such as numerous renewals with little or no repayment of principal, new loans or renewals with additional advances, and questionable collateral protection... Management extended new loans that were used to make payments on existing loans and to cover current taxes and insurance. First NBC also extended loans and allowed proceeds to be used to pay off other delinquent bank loans, again without any requirement for principal payments from the borrowers."

[2]Banerjee and Hofmann (2018) also argue that low interest rates as opposed to weak bank capital contribute to the rise of zombie lending. However, the channels though which low interest rates operate are largely unexplored.

[3]Sometimes, people also refer to "zombie lending" as "extend and pretend" or "evergreening". They all refer to the decisions to lend to borrowers that are known to be in distress.

conduct zombie lending for loans that turn out to be bad later on, because these bad loans can be pooled with good ones.

To be more specific, we model an entrepreneur that invests in a long-term, illiquid project whose quality is either good or bad. A good project should continue to be financed, whereas a bad project should be immediately liquidated. Initially, the quality of the project is unknown to everyone, including the entrepreneur. The entrepreneur can raise funding from either the competitive financial market or a bank. Market financing takes the form of arm's-length debt, so lenders only need to break even given their beliefs about the project's quality. Bank lending, in contrast, will develop into a relationship. Under market financing, no information is ever produced, whereas the screening and monitoring associated with bank lending produce "news" about the project's quality. We model news arrival as a Poisson event and assume that it is observed only by the entrepreneur and the bank, that is, the bank and the entrepreneur *privately* learn the project's quality as time goes by. Meanwhile, all agents, including lenders in the financial market, can observe the time since the initialization of the project, which will turn out to be the important state variable. When the bank loan matures, the bank and the entrepreneur decide to roll it over, to liquidate the project, or to refinance with market-based lenders. This decision depends crucially on the level of the state variable and is the central focus of the paper.

We show that equilibrium is characterized by two thresholds in time and therefore comprises three stages. In the first stage, a project is liquidated upon learning bad news, whereas other loans will be rolled over. During this period, the average quality of borrowers who remain with banks improves. Equivalently, borrowers that remain with the bank gain reputation from the liquidation decisions of the bad types. These liquidation decisions are socially efficient, and thus we name this stage *efficient liquidation*. In the second stage, all loans will be rolled over irrespective of their quality. In particular, the relationship bank will roll over the loan even if it knows that the project is bad –this bank keeps *extending* the loan to *pretend* no bad news has occurred, which is inefficient. This result of banks rolling over bad loans can be interpreted as zombie lending. Finally, in the last stage, all entrepreneurs refinance with the market upon their bank loans maturing. We refer to this stage as the *market financing* stage.

The intuition for these results is best explained by looking backwards in time. When elapsed time gets sufficiently long, all entrepreneurs will become sufficiently reputable to switch to market financing, as we assume that market-based lenders are competitive and offer lower costs of capital. This outcome is the equilibrium in the last stage. Now imagine that bad news arrives shortly before the last stage. The relationship bank could liquidate the project, in which case it receives a low liquidation value. Alternatively, it can roll over the loan and pretend that no bad news has arrived yet. By hiding bad news today, the bank helps the borrower maintain its reputation in order to refinance with the market in the future. Such zombie lending dominates liquidation, because the bank will be fully repaid at the time of market refinancing. In this case, the expected loss will likely be borne by the market-based lenders. By contrast, if negative news arrives early, zombie lending is much more costly to the bank, due to both large time discounting and a high probability that the project may mature before the arrival of the last stage, in which case the expected loss will be borne by the relationship bank. Liquidating the project is therefore preferred.

Our equilibrium highlights three sources of inefficiency relative to the first-best benchmark. First, as in a standard dynamic lemons problem, a good borrower experiences a delay in receiving market financing. Second, a bad borrower is no longer liquidated after the first stage, even though liquidation has a higher social value. Finally, an uninformed-type borrower refinances with the market in the third stage, which is too soon compared to the first-best benchmark. Note this last source of inefficiency is contrary to that in the dynamic lemons problem, as the inefficiency is not the existence of delay but rather insufficient delay.

We show that the concern for zombie lending is mitigated under a financial constraint, which essentially limits the repayments from the borrower to the bank. In particular, this constraint leads to scenarios in which a bad project is liquidated, even though the liquidation value falls below the joint surplus if both parties choose to roll it over. As a result, the efficient liquidation period becomes longer and the zombie lending period becomes shorter.

Our interpretation of learning is the bank screening and monitoring process, which generates useful information about the entrepreneur's business prospects but cannot be shared with others in the financial market. When we endogenize learning as a costly decision, we show the bank ceases to learn during the efficient liquidation stage. Intuitively, the benefit of learning arises because an informed bad bank could liquidate a bad project for the liquidation value. This learning benefit vanishes after time passes the efficient liquidation stage. This result highlights a new type of hold-up problem in a lending relationship: the bank underinvests in producing information when it anticipates that the borrower will refinance with the market in the future. Note that this result holds even if the relationship bank has all of the bargaining power, because it is unable to capture all of the surplus – including current and future surplus – generated from learning.

Our paper is consistent with existing empirical evidence and anecdotal stories. Moreover, the result on zombie lending offers new testable implications. First, the age distribution of liquidated loans should be left-skewed, with loan renewals containing more favorable terms over time. Second, our interpretation of the market-financing stage includes debt initial public offerings, loan sales and securitizations, and anticipated credit rating upgrades. Our model thus predicts that the positive announcement effect associated with loan renewals should be small or even zero if any of these events happens shortly after renewal. More broadly, our result implies that the development of financial markets, such as loan sales and securitizations, as well as improvement in bond market liquidity can exacerbate zombie lending.

Related Literature

Broadly, our paper is related to three stands of literature. We build on the approach of dynamic signaling and private learning (Janssen and Roy (2002), Kremer and Skrzypacz (2007), Daley and Green (2012), Fuchs and Skrzypacz (2015), Grenadier, Malenko, and Strebulaev (2014), Atkeson, Hellwig, and Ordoñez (2014), Marinovic and Varas (2016), Martel, Mirkin, and Waters (2018), Hwang (2018), Kaniel and Orlov (2020)). In our model, news is private, whereas in Daley and Green (2012), news is publicly observable.[4] Martel, Mirkin, and Waters (2018) and Hwang (2018) also study problems in which sellers become gradually informed about an asset's quality. Besides the specific application to relationship banking, our model has different theoretical implications. First, sellers in these two papers only choose the time of trading, whereas in our model the bank is also endowed with the option to liquidate.[5] This additional option, which is natural in the banking context, generates different dynamics and efficiency implications. In our paper, bad types initially choose to separate through gradual liquidation and only pool with other types after their reputation is sufficiently high. Moreover, whereas delayed trading is always inefficient in these papers, our paper additionally highlights insufficient delay for uninformed types and lack of liquidation for bad types. Second, we study a problem in which learning is costly and endogenous and show how reputation and asymmetric information affect learning incentives. In doing so, we discover a new type of hold-up problem in banks' information production.

Our paper is among the first to introduce dynamic learning in the context of banking (also see Halac and Kremer (2020) and Hu (2021)). We extend previous work in relationship banking by Diamond (1991b), Rajan (1992), Boot and Thakor (2000), and Parlour and Plantin (2008), among others, by studying the impact of dynamic learning and adverse selection on lending relationships. Whereas Diamond (1991a) emphasizes reputation buildup during bank lending, borrowers are financed with arm's-length debt and lenders' decisions are myopic, implying that lenders will never have incentives to roll over bad loans. Rajan (1992) studies the trade-off between relationship-based lending and arm's-length debt, without an explicit role for the borrower's reputation. Chemmanur and Fulghieri (1994a,b) emphasize the role of lenders' reputation in borrower choices between bank versus market financing, whereas our paper emphasizes borrowers' reputation. Parlour and Plantin (2008) study the secondary market, in which a bank may

[4] Our model also has a public news process to justify the off-equilibrium belief.

[5] The bank and the entrepreneur can be thought of as the seller, whereas market-based lenders are buyers.

sell loans if a negative capital shock arises or if the loan is privately known to be bad. They show that a liquid secondary market reduces a bank's incentive to monitor. Our paper focuses on the dynamics of loan rollover and studies dynamic reasons for banks to sell loans. Specifically, the adverse selection concern is endogenously buildup over time and depends on the borrower's reputation. Bolton et al. (2016) study the choice between transaction and relationship banking under a similar assumption, whereby the relationship bank has a higher cost of capital but is able to learn the borrower's type. The authors show that borrowers are willing to pay the relationship bank higher interest rates during normal times in order to secure funding during crises. Our paper has a different focus, showing that the superior information acquired by the relationship bank can result in inefficient zombie lending.

Another literature adopts a dynamic contracting approach to study relationship lending. Boot and Thakor (1994) show that a long-term credit contract allows the lender to use future low interest so that the equilibrium contract does not involve collateral once the borrower successfully repays a single-period loan. This implies that collateral usage will decline as relationship duration increases. Verani (2018) builds a quantitative general-equilibrium model and shows that if the borrower has limited commitment, the lender is willing to accept delayed credit payments in exchange for higher continuation values. Sanches (2010) similarly shows that the optimal dynamic contract features delayed settlement and debt forgiveness. Note that delayed payment and forgiveness are necessary for borrowers to remain in the lending relationship and repay in the future. Both features are different from zombie lending in our model, where lenders roll over credit to cover bad private news.[6]

Our explanation for zombie lending differs from existing theories that rely largely on regulatory capital requirements (Caballero, Hoshi, and Kashyap (2008), Peek and Rosengren (2005)). Rajan (1994) uses a signal-jamming model and explains the phenomenon of rolling over bad loans by assuming that myopic loan officers face career concerns. In this literature, terminating a bad loan results in a negative shock to bank capital, which can trigger regulatory actions including bank closure (e.g., Kasa and Spiegel (1999)). This can make banks reluctant to recognize losses by writing off bad loans. In our paper, banks are well capitalized and zombie lending emerges in equilibrium because banks are forward-looking instead of myopic. In this sense, our explanation, based on borrowers' reputation, complements existing ones. Similarly, Puri (1999) shows that banks have incentives to certify a bad firm, hoping that investors will invest and repay the loan. Her explanation focuses on the lender's reputation, whereas our paper highlights the importance of the borrowing firm's reputation. Our paper is also related to previous work

[6]The reason the relationship bank does not liquidate the borrower is fundamentally different. In the dynamic contracting literature, the bank chooses not to liquidate in order to incentivize the borrower to *remain* in the relationship. In our paper, the bank chooses not to liquidate in order to incentivize the bad borrower to *leave* the relationship by refinancing with others in the future.

on debt rollover by He and Xiong (2012), Brunnermeier and Oehmke (2013), He and Milbradt (2016), and particularly to Geelen (2019), who models the dynamic tradeoff of debt issuance and rollover under asymmetric information. In contrast to this literature, which focuses largely on competitive lenders, we model one lender that becomes gradually informed – the bank – together with competitive lenders – the market.

I. Model

We consider a continuous-time model with an infinite horizon. An entrepreneur invests in a long-term project with unknown quality. She borrows from either a bank, which will develop into a relationship, or the competitive financial market. Compared to market financing, bank financing has the advantage of producing valuable information but with the downside of a higher cost of capital and the possibility of information monopoly. Below, we describe the model in detail.

A. Project

We consider a long-term project that generates a constant stream of interim cash flows cdt over a period $[t, t + dt]$. The project matures at a random time τ_ϕ, which arrives at an exponential time with intensity $\phi > 0$. Upon maturity, the project produces random final cash flows, depending on its type. A good (g) project produces cash flows R with certainty, whereas a bad (b) project produces R with probability $\theta < 1$. With probability $1 - \theta$, a matured bad project fails to produce any final cash flows. In addition to failing to generate final cash flows, a bad project may fail prematurely, in which case it stops generating any cash flows, including both interim cash flows and final cash flows. The premature failure event arrives at an independent exponential time τ_η, where $\eta \geq 0$ is the arrival intensity. We sometimes refer to this premature failure as *public* news. We assume that η is sufficiently low and can be zero, so that none of the main results depend on this public news process.

Initially, no agent, including the entrepreneur herself, knows the project's type – all agents share the same public belief that q_0 is the probability of the project being good. If the project fails prematurely, all agents will learn that the project is bad with certainty. At any time before the final cash flows are produced or premature failure occurs, the project can be terminated with liquidation value $L > 0$. In Assumption 1 below, we impose the parametric assumption that L is higher than the value of discounted future cash flows generated by a bad project. Therefore, liquidating a bad project will be socially valuable. Note that the liquidation value is independent of the project's quality, so it should be understood as the liquidation of the physical asset used in production. For example, one can think of L as the value of the asset if redeployed (Benmelech (2009)).

Let $r > 0$ be the entrepreneur's discount rate. The fundamental value of the project to the entrepreneur at $t = 0$ is therefore given by the discounted value of its future cash flows,

$$PV_r^g = \frac{c + \phi R}{r + \phi}, \quad PV_r^b = \frac{c + \phi\theta R}{r + \phi + \eta}, \quad PV_r^u = q_0 PV_r^g + (1 - q_0) PV_r^b. \tag{1}$$

Note that the denominator of NPV_r^b contains an additional term η, which accounts for the premature failure event.

REMARK 1. Although we do not explicitly model the initial investment, one can imagine that a fixed investment scale I is needed at $t = 0$ to initialize the project. In section C.1, we derive the maximum amount that an entrepreneur is able to raise at the initial date. The project is not initialized if this amount falls below I.

B. Agents and Debt Financing

The borrower has no wealth and needs to borrow through debt contracts. The use of debt contracts is not crucial and can be justified by nonverifiable final cash flows (Townsend (1979)). One can also interpret these contracts as equity shares with different control rights and therefore think of the entrepreneur as a manager of a start-up venture. We consider two types of debt, that offered by banks and that offered by market-based lenders. First, the entrepreneur can take out a loan from a banker, who has the same discount rate r. For tractability reasons, we assume that a bank loan lasts for a random period and matures at a random time τ_m, upon the arrival of an independent Poisson event with intensity $\frac{1}{m} > 0$. The parameter m can be interpreted as the expected maturity of the loan. In most of the analysis, we study the limiting case of instantly maturing loans, that is, $m \to 0$. Section II.D solves the case for general m, and shows that the results are qualitatively unchanged.

The second type of debt is provided by the market. One can think of this debt as public bonds. We consider a competitive financial market in which lenders have discount rate δ satisfying $\delta < r$. This assumption implies that market financing is cheaper than bank financing. We define the value of the project to the market as

$$PV_\delta^g = \frac{c + \phi R}{\delta + \phi}, \quad PV_\delta^b = \frac{c + \phi\theta R}{\delta + \phi + \eta}, \quad PV_\delta^u = q_0 PV_\delta^g + (1 - q_0) PV_\delta^b. \tag{2}$$

The assumption $\delta < r$ captures the realistic feature that banks have a higher cost of capital than the market, which can be justified by either regulatory requirements or the skin in the game needed to monitor borrowers (see Holmstrom and Tirole (1997); see also Schwert (2018) for recent empirical evidence). As we clarify shortly, the maturity of the public debt does not matter. For simplicity, we assume that the public debt always matures with the project.

Both types of debt share the same exogenously specified face value: $F \in (L, R)$. The condition $F > L$ guarantees debt is risky, whereas $F < R$ captures the wedge between a project's maximum income and its pledgeable income (Holmström and Tirole (1998)).[7] All of our results will go through if $F \equiv R$ but some nonpledgeable control rents accrue to the entrepreneur if the project matures. Note that we take F as given: we aim to study the trade-off between relationship borrowing and public debt, rather than the optimal leverage. At $t = 0$, the entrepreneur chooses between public debt and a bank loan that will develop into a relationship. Once the bank loan matures, the entrepreneur can still replace it with a public bond. Alternatively, she could roll over the loan with the same bank, which may have an information advantage over the project's quality.[8] In this case, the two parties bargain over y_t, the interest rate of the loan until the next rollover date. The financial constraint that the entrepreneur has no wealth restricts y_t to be weakly less than c, the level of the interim cash flows. In the remainder of this paper, we assume that the bank always has all of the bargaining power. The results under interior bargaining power will differ only quantitatively. The allocation of the bargaining power together with the financial constraint $y_t \leq c$ naturally leads to the result that $y_t \equiv c$. As we show below, this financial constraint limits the size of the repayment that the entrepreneur can make to the bank, and thus the Nash bargaining outcome is sometimes not the one that maximizes the joint surplus of the two parties.

Because market financing is competitive and market-based lenders have a lower cost of capital, the entrepreneur will always prefer to take the highest leverage possible once she borrows from the market. The coupon payments associated with the public bond are therefore equal to $c\,dt$.

REMARK 2. We assume that the entrepreneur is allowed to take only one type of debt. In other words, we rule out the possibility of the entrepreneur using a more sophisticated capital structure to signal her type. See Leland and Pyle (1977) and DeMarzo and Duffie (1999) for discussion of these issues.

C. *Learning and Information Structure*

The quality of the project is initially unknown, with $q_0 \in (0, 1)$ being the commonly shared belief that it is good. If the entrepreneur finances with the bank, that is, if she takes out a loan, the entrepreneur-bank pair can *privately* learn the quality of the project through "news." Private news arrives at a random time τ_λ, modeled as an independent

[7]The maximum pledgeable cash flow can be microfounded by some unobservable action taken by the entrepreneur (e.g., cash diversion) shortly before the final cash flows are produced (Tirole (2010)).

[8]We assume without loss of generality that the entrepreneur would never switch to a different bank upon loan maturity. Intuitively, the market has a lower cost of capital than an outsider bank and the same information structure.

Poisson event with intensity $\lambda > 0$. Upon arrival, the news perfectly reveals the project's type. In practice, one can think of the news process as information learned during bank screening and monitoring. We assume that such news can be observed only by the two parties and that no committable mechanism is available to share it with third parties, such as credit bureaus and market participants. In this sense, the news can be understood as soft information on project quality (Petersen, 2004)). For instance, one can think of this news as the information that banks acquire upon due diligence and covenant violation, which includes details on the business prospect, collateral quality, and financial soundness of the borrower. In the benchmark model, we take the learning of private news as exogenous. Section III solves a model in which learning incurs a physical cost. We show that the bank will incur this cost only in the early stage of a lending relationship.

Although public market participants do not observe the private news, they can observe (1) the public news – whether the project has failed prematurely, (2) t – the project's time since initialization, and (3) whether the project has been liquidated. Therefore, the public can infer the project's quality based on these observations. Let $i \in \{u, g, b\}$ denote the type of the bank/entrepreneur, where u, g, and b refer to the uninformed, informed-good, and informed-bad types, respectively. Let μ_t be the (naive) belief about the project's quality if the market lenders learn solely from the fact that the project has not failed prematurely. A standard filtering result implies that

$$\dot{\mu}_t = \eta \mu_t \left(1 - \mu_t\right), \tag{3}$$

where $\mu_0 = q_0$. Note that the public news could only be bad, which occurs if the project fails prematurely.

We first describe the *private* belief process, that is, the belief held by the bank and the entrepreneur. If the private news has not arrived yet, the private belief remains at μ_t. Upon news arrival at t_λ, the private belief jumps to one in the case of good news and to zero in the case of bad news. To characterize the public belief process, we introduce a belief system $\left\{\pi_t^u, \pi_t^g, \pi_t^b\right\}$, where π_t^u is the public's belief at time t that the private news has not arrived yet and π_t^g (π_t^b) is the public's belief that the private news has arrived and is good (bad). In any equilibrium in which the belief is rational, π_t^i is consistent with the actual probability that the bank and the entrepreneur are of type $i \in \{u, g, b\}$. Given $\left\{\pi_t^u, \pi_t^g, \pi_t^b\right\}$, the public belief that the project is good is[9]

$$q_t = \pi_t^u \mu_t + \pi_t^g. \tag{4}$$

In the remainder of this paper, we sometimes refer to q_t as the average quality or the average belief.

[9]To simplify notation, we abuse notation and use $\left\{\pi_t^i, q_t\right\}$ to denote $\left\{\pi_{t-}^i, q_{t-}\right\}$.

REMARK 3. Note that learning and the arrival of private news require input from both the entrepreneur and the bank. We can therefore think of learning as exploration of the underlying business prospect, which requires the entrepreneur's experimentation and the bank's previous experience in financing related businesses. In this sense, our model could also be applied to study venture capital firms. Alternatively, we can interpret learning as a process that relies solely on the entrepreneur's input, which is independent of the source of financing, whereas only the bank observes the news obtained through monitoring. Put differently, even without bank financing, the entrepreneur is able to learn about the quality of her project over time. Our results in Section II are identical in this alternative setting, because in the lending relationship, the bank and the entrepreneur are always equally informed.

D. Rollover

When the loan matures, the entrepreneur and the bank have three options: liquidate the project for L, switch to market financing, or continue the relationship by rolling over the loan. Control rights are assigned to the bank if the loan is not fully repaid, and renegotiation could potentially be triggered. Let $O_t^i \equiv O_{Et}^i + O_{Bt}^i,\ i \in \{u, g, b\}$, be the maximum joint surplus to the two parties if the loan is not rolled over, where O_{Et}^i and O_{Bt}^i are the values that accrue to the entrepreneur and the bank, respectively. Because $F > L$, in the case of liquidation, the bank receives the entire liquidation value L and the entrepreneur receives nothing, that is, $O_{Bt}^i = L$ and $O_{Et}^i = 0$. If the two parties are able to switch to market financing, the bank receives full payment $O_{Bt}^i = F$ and the entrepreneur receives the remaining surplus $O_{Et}^i = \bar{V}_t^i - F$, where

$$\bar{V}_t^g = D_t + \frac{\phi (R - F)}{r + \phi}, \qquad \bar{V}_t^b = D_t + \frac{\phi\theta (R - F)}{r + \phi + \eta}, \qquad \bar{V}_t^u = \mu_t \bar{V}_t^g + (1 - \mu_t) \bar{V}_t^b. \quad (5)$$

In (5),

$$D_t = \hat{q}_t D^g + (1 - \hat{q}_t) D^b \quad (6)$$

is the competitive price of a bond at time t, where $D^g = \frac{c+\phi F}{\delta+\phi}$ and $D^b = \frac{c+\phi\theta F}{\delta+\phi+\eta}$ are the price of the bond for a good-type and bad-type project, respectively, and $\hat{q}_t$ is the average quality of the project conditional on refinancing with the market. In the case in which all types choose to refinance, $\hat{q}_t = q_t$. The second terms in (5) are the discounted value of the final cash flows that the entrepreneur $i \in \{u, g, b\}$ receives upon the project's maturity.

Two conditions need to be satisfied for a loan to be rolled over. First, $V_t^i > \max\left\{L, \bar{V}_t^i\right\}$, so that rolling over indeed maximizes the joint surplus. Second, because the interest rate of the loan y_t cannot go beyond c, the bank needs to prefer rolling over the loan with interest rate c to liquidating the project for L.

E. Strategies and Equilibrium

The public history $\mathcal{H}_t$ consists of (1) time t, (2) whether the project has failed prematurely, and (3) the actions of the entrepreneur and the bank up to time t. Specifically, the action set includes for any time $s \le t$ whether the entrepreneur borrows from the bank or the market and whether the project has been liquidated. For any public history, the price of market debt D_t summarizes the market lender's strategy. Given that the market is competitive, the price of debt satisfies (6).

The private history h_t consists of the public history $\mathcal{H}_t$, the rollover event, the Poisson event on private news arrival, and the content of the news. Essentially, the strategy of the entrepreneur and the bank is to choose an optimal stopping time, and at the stopping time, whether to liquidate the project or refinance with the market. This choice is subject to the additional constraint that at the stopping time, the bank's continuation value is at least (weakly) greater than L, the liquidation value of the project. Let V_t^i be the joint value of the entrepreneur and the bank in the lending relationship, B_t^i be the continuation value of the bank,[10] and τ^i be the (realized) stopping time of type i $i \in \{u, g, b\}$.[11] We then have

$$\begin{aligned} V_t^u = \max_{\substack{\tau^u \ge t, \\ s.t.\ B_{\tau^u}^u \ge L}} \quad & \mathbb{E}_{t^-}\Bigg\{ \int_t^{\tau^u} e^{-r(s-t)} c ds + e^{-r(\tau^u - t)} \Bigg[\mathbb{1}_{\tau^u \ge \tau_\phi} \left[\mu_{\tau_\phi} + \left(1 - \mu_{\tau_\phi}\right) \theta\right] R + \mathbb{1}_{\tau^u \ge \tau_\eta} \cdot 0 \\ & + \mathbb{1}_{\tau^u \ge \tau_\lambda} \left[\mu_{\tau_\lambda} V_{\tau_\lambda}^g + (1 - \mu_{\tau_\lambda}) V_{\tau_\lambda}^b\right] + \mathbb{1}_{\tau^u < \min\{\tau_\phi, \tau_\lambda, \tau_\eta\}} \max\{L, \bar{V}_{\tau^u}^u\} \Bigg] \Bigg\}, \end{aligned} \tag{7}$$

[12]

and

$$\begin{aligned} B_t^u = & \mathbb{E}_{t^-}\Bigg\{ \int_t^{\tau^u} e^{-r(s-t)} c ds + e^{-r(\tau^u - t)} \Bigg[\mathbb{1}_{\tau^u \ge \tau_\phi} \left[\mu_{\tau_\phi} + \left(1 - \mu_{\tau_\phi}\right) \theta\right] F + \mathbb{1}_{\tau^u \ge \tau_\eta} 0 \\ & + \mathbb{1}_{\tau^u \ge \tau_\lambda} \left[\mu_{\tau_\lambda} B_{\tau_\lambda}^g + (1 - \mu_{\tau_\lambda}) B_{\tau_\lambda}^b\right] + \mathbb{1}_{\tau^u < \min\left\{\tau_\phi, \tau_\lambda, \tau_\eta\right\}} \max\left\{L, \min\left\{\bar{V}_{\tau^u}^u, F\right\}\right\} \Bigg] \Bigg\}. \end{aligned} \tag{8}$$

In (7), τ^u is the stopping time of the entrepreneur and the bank if both are uninformed. The first term, $\int_t^{\tau^u} e^{-r(s-t)} c ds$, is the value of interim cash flows until τ^u. The project matures and pays off the final cash flows if $\tau^u \ge \tau_\phi$. If $\tau^u \ge \tau_\eta$, the project fails

[10] We use the standard notation $\mathbb{E}_{t-}[\cdot] = \mathbb{E}[\cdot | h_{t-}]$ to indicate that the expectation is conditional on the history before the realization of the stopping time τ.

[11] Formally, let t^u be the optimal stopping time to liquidate or refinance chosen by type u. Then $\tau^u = \min\{\mathrm{t}^u, \tau_\phi, \tau_\eta, \tau_\lambda\}$. Stopping times τ^g and τ^b can be similarly defined.

[12] In the model with general maturity $m > 0$, τ_u is restricted to the set of the rollover dates.

prematurely, with the continuation payoff equal to zero. If $\tau^u \geq \tau_\lambda$, private news arrives, after which the two parties become informed. Finally, if $\tau^u < \min\{\tau_\phi, \tau_\eta, \tau_\lambda\}$, the bank and the entrepreneur choose to stop before any of the above events arrives, and they decide whether to liquidate the project for L or refinance with the market for $\bar{V}^u_{\tau^u}$. The decision is made subject to the constraint that $B^u_{\tau^u} \geq L$. Equation (8) can be interpreted similarly. The value functions of types g and b are similarly defined in the Appendix.

We look for a perfect Bayesian equilibrium of this game.

DEFINITION 1: *An equilibrium of the game satisfies the following conditions:*

1. *Optimality: The rollover decisions are optimal for the bank and the entrepreneur, given the belief processes* $\{\pi^i_t, \mu_t, q_t\}$.

2. *Belief Consistency: For any history on the equilibrium path, the belief process* $\left\{\pi^u_t, \pi^g_t, \pi^b_t\right\}$ *is consistent with Bayes' rule.*

3. *Market Breakeven: The price of the public bond satisfies* (6).

4. *No (Unrealized) Deals:*[13] *For any* $t > 0$ *and* $i \in \{u, g, b\}$,

$$V^g_t \geq \mathbb{E}\left[D^i|\mathcal{H}_t,\ D^i \leq D^g\right] + \frac{\phi\,(R-F)}{r+\phi}$$
$$V^u_t \geq \mathbb{E}\left[D^i|\mathcal{H}_t,\ D^i \leq D^u\right] + \mu_t\frac{\phi\,(R-F)}{r+\phi} + (1-\mu_t)\frac{\phi\theta\,(R-F)}{r+\phi+\eta},$$

where

$$D^u = \mu_t D^g + (1-\mu_t)\,D^b.$$

5. *Belief Monotonicity: Continued bank financing is never perceived as a (strictly) negative signal,* $\dot{q}_t \geq \eta q_t\,(1-q_t)$.

The first three conditions are standard. The No-Deals condition follows Daley and Green (2012), reflecting the requirement that the market cannot profitably deviate by making an offer that the entrepreneur and the bank will accept. Note that the second terms on the right-hand side of the No-Deals condition reflect the fact that even after market refinancing, the entrepreneur's continuation payoff is still type-specific.

As is standard in the literature, we use a refinement to rule out unappealing equilibria that arise due to unreasonable beliefs. Specifically, we impose a belief monotonicity refinement whereby continued bank financing is never perceived as a (strictly) negative signal. As a result, the public belief about the project's quality conditional on bank

[13] We offer a micro foundation as follows. In each period, two short-lived market-based lenders simultaneously enter and make private offers to all entrepreneurs. This microfoundation will give rise to the *No-Deals* condition as in Daley and Green (2012).

financing is weakly higher than the naive belief process that is updated only from the public news that no premature failure has occurred yet. In effect, this condition eliminates equilibria that can arise due to threatening beliefs. For example, suppose the belief is that a project that does not refinance with the market at time $\hat{t}$ is treated as a bad type. Then under some conditions, all types will be forced to refinance at time $\hat{t}$.

F. Parametric Assumptions

To make the problem interesting, we make the following parametric assumptions.

ASSUMPTION 1: *(Liquidation Value):*

$$PV_{\delta}^{b} < L < \frac{\delta+\phi}{r+\phi} D^{g} + \frac{\phi\theta\,(R-F)}{r+\phi}. \tag{9}$$

The first half of Assumption 1 says that the liquidation value L is above the discounted cash flows of a bad project to the market. Therefore, liquidating a bad project is socially optimal. The second half assumes that if a bad-type borrower can refinance with the market at the price of a good-type's bank debt, it will not liquidate the project. Note that this assumption implies $L < PV_{r}^{g}$, so continuing a good project is socially optimal. In the absence of the liquidation option, the equilibrium results are straightforward. In particular, all types of borrowers will immediately finance with the market at $t = 0$.[14] As we will see in the next section, this result is no longer true with the option to liquidate.

ASSUMPTION 2: *(Risky Loan):*

$$F > \max\left\{\theta R, L, D^{b}\right\}. \tag{10}$$

Assumption 2 assumes that the face value of the debt is above the liquidation value, the expected repayment, and the price of the bond of a bad project; otherwise, the loan is effectively riskless.

ASSUMPTION 3: *(Interim Cash Flow):*

$$c \geq rF. \tag{11}$$

Assumption 3 guarantees that the size of the interim cash flow c is large enough to cover the lenders' cost of capital. Otherwise, the face value of the loan F needs to grow during rollover dates.

[14] The proof follows directly from applying the Law of Iterated Expectation and the assumption that bank financing is more costly.

ASSUMPTION 4: *(Optimal Bank Financing):*

$$D^b < \frac{\delta + \phi}{r + \phi} D^g \tag{12}$$

$$PV^b_\delta < \frac{c + \phi\theta R + \lambda L}{r + \phi + \lambda + \eta}. \tag{13}$$

This assumption imposes restrictions so that at least some level of bank financing will be used in the first-best benchmark. Equation (12) is the static lemons condition in the literature (Daley and Green (2012), Hwang (2018)), which requires that the price of a bad-type bond be lower than the value of a good-type loan. Given Assumption 1, (13) essentially requires that λ be sufficiently high that the private news produced during bank financing is sufficiently useful.

The first-best outcome is achieved if the private news can be publicly observable.

PROPOSITION 1: *A unique pair* $\{\underline{\mu}_{FB}, \bar{\mu}_{FB}\}$ *exists such that in the first-best benchmark,*

1. *If* $q_0 \leq \underline{\mu}_{FB}$*, the unknown project is liquidated at* $t = 0$*.*

2. *If* $q_0 \in (\underline{\mu}_{FB}, \bar{\mu}_{FB})$*, the unknown project is financed with the bank at* $t = 0$*.*

3. *If* $q_0 \geq \bar{\mu}_{FB}$*, the unknown project is financed with the market at* $t = 0$*.*

Assumption 1 leads to the result that any good project will immediately receive financing from the market, whereas a bad project will be liquidated upon news arrival. According to Proposition 1, an unknown project with belief $q_0 \in (\underline{\mu}_{FB}, \bar{\mu}_{FB})$ should start with bank financing due to the option value of information. Over time, either news or the premature failure event may arrive, at which point the project receives immediate market financing following good news and is immediately liquidated following bad news. In the absence of news and premature failure, the belief about the project follows (3). In this case, the project will be financed with the market once μ_t reaches $\bar{\mu}_{FB}$.

In the remainder of this paper, we assume that $q_0 \in (\underline{\mu}_{FB}, \bar{\mu}_{FB})$.

II. Equilibrium

We solve the model in this section. In Section II.A, we study an economy without the financial constraint that the interest rate on the loan satisfies $y_t \leq c$. The main result is that a zombie lending region $[t_b, t_g]$ exists over which the bank will always roll over the loan, even if it has already learned that the borrower's project is bad. Section II.B studies the equilibrium with a formal treatment of the financial constraint $y_t \leq c$. We show that the equilibrium structure is similar to that in Section II.A, but the constraint reduces the length of the zombie lending region. We present a special case without premature failure in Section II.C, where all results are derived in simple and closed form. Section

II.D further extends the analysis to loans with general maturity and studies the effect of loan maturity.

A. *Benchmark Without the Financial Constraint* $y_t \leq c$

The benchmark case without the financial constraint $y_t \leq c$ essentially assumes a deep-pocketed entrepreneur. In particular, the entrepreneur could borrow a loan with interest rate $y_t > c$. Given the Nash bargaining assumption at each rollover date, we can treat the bank and the entrepreneur as one entity, where the problem of the entity is to choose two optimal stopping times. First, it decides when to liquidate the project. Second, it decides when to switch to market financing by replacing the loan with public debt.

The economy is characterized by state variables in private and public beliefs. All public beliefs (without liquidation and public news) turn out to be deterministic functions of the elapsed time. We therefore use time t as the state variable. Specifically, we construct an equilibrium characterized by two thresholds $\{t_b, t_g\}$, as illustrated by Figure 1. If $t \in [0, t_b]$, the bank and the entrepreneur will liquidate the project upon the arrival of bad news – *efficient liquidation* region. Loans for other project types (good and unknown) will be rolled over. If $t \in [t_b, t_g]$, all types of loans will be rolled over, including bad ones – *zombie lending* region. Finally, if $t \in \left[t_g, \infty\right)$, the two entities will always refinance with the market upon loan maturity – *market financing* region.[15]

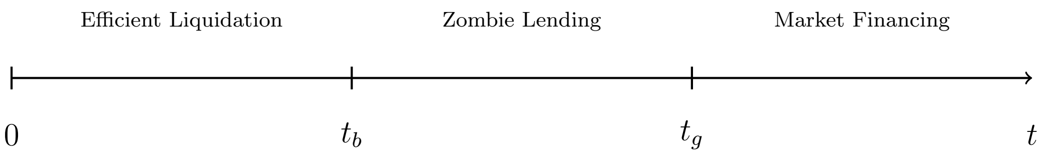


Figure 1. Equilibrium regions.

Given the equilibrium conjecture, the evolution of beliefs follows Lemma 1.

LEMMA 1: *In an equilibrium with thresholds* $\{t_b, t_g\}$*, the belief about a project's average quality evolves according to*

$$\dot{q}_t = \begin{cases} (\lambda + \eta)\, q_t \,(1 - q_t) & t \leq t_b \\ \eta q_t \,(1 - q_t) & t > t_b \end{cases} \tag{14}$$

with initial condition q_0.

Heuristically, before t reaches t_b, q_t evolves as if the premature failure arrives at rate $\lambda + \eta$, because a project will be immediately liquidated following bad private news. After

[15] With instantly maturing loans, all banks and entrepreneurs will refinance with the market immediately at t_g. In the case with general maturity, the market financing region is $\left[t_g, \infty\right)$, depending on when the bank loan matures.

t reaches t_b, however, q_t evolves as if no private news exists at all, because a privately known bad project will no longer be liquidated.

Next, we characterize the continuation value in different equilibrium regions, as well as the boundary conditions. To better explain the economic intuition, we describe the results backwards in the elapsed time.

Market Financing: $\{t_g\}$. In this region, $V_t^i = \bar{V}_t^i$, $i \in \{u, g, b\}$, where $\left\{\bar{V}_t^u, \bar{V}_t^g, \bar{V}_t^b\right\}$ are as defined in (5) with $\hat{q}_t = q_t$. The economic intuition is as follows. Ultimately, if the entrepreneur's reputation becomes sufficiently high, market financing is cheaper because market lenders are competitive and associated with a lower cost because $\delta < r$. As a result, all types will replace their loans with public bonds. The threshold in reputation is obtained as the public belief q_t increases to $\bar{q}$. As we show below, this increase arises because in equilibrium, bad types would have failed prematurely or been liquidated. The absence of both premature failure and liquidation helps the entrepreneur accumulate reputation.

Zombie Lending: $\left[t_b, t_g\right)$. Working backwards, we now consider the region $\left[t_b, t_g\right)$ over which all types of loans, including bad ones, are rolled over. Mathematically, the value functions of all three types satisfy the following Hamilton-Jacobi-Bellman (HJB) equation system:

$$(r + \phi + \lambda + (1 - \mu_t)\,\eta)\,V_t^u = \dot{V}_t^u + c + \phi\,[\mu_t + (1 - \mu_t)\,\theta]\,R + \lambda\left[\mu_t V_t^g + (1 - \mu_t)\,V_t^b\right] \tag{15a}$$

$$(r + \phi)\,V_t^g = \dot{V}_t^g + c + \phi R \tag{15b}$$

$$(r + \phi + \eta)\,V_t^b = \dot{V}_t^b + c + \phi\theta R. \tag{15c}$$

The first term on the right-hand side of (15a) is the change in valuation due to time, the second term captures the benefits of interim cash flow, and the third term corresponds to the event of project maturity, which arrives at rate ϕ. In this case, the bank and the entrepreneur receive final cash flows R with probability $\mu_t + (1 - \mu_t)\,\theta$. The fourth term stands for the arrival of private news at rate λ. Following the news, the bank and the entrepreneur become informed. Equations (15b) and (15c) can be interpreted in a similar vein.

When time gets close to t_g, the bank and the entrepreneur find that waiting until t_g and refinancing with the market is optimal, even if bad news has arrived. Intuitively, rolling over bad loans allows the bank to be fully repaid at t_g. When time is close to t_g, this decision can be optimal compared to liquidating the project for L. In this region, even though no project is liquidated, the entrepreneur's reputation keeps growing as long as the project does not fail prematurely.

We show that $t_g - t_b > 0$, which implies that zombie lending is inevitable in a dynamic lending relationship. Equilibrium in this region is clearly inefficient. A bad project should be liquidated, but instead the bank and the entrepreneur roll it over in the hope of passing the losses onto market lenders at t_g. As we see next, by not liquidating between $t = 0$ and t_b, they accumulate a good reputation and thus zombie lending can be sustained in equilibrium.

Efficient Liquidation: $\left[0, t_b\right)$. Finally, we turn to the first region $\left[0, t_b\right)$, where bad loans are not rolled over but instead liquidated. Mathematically, V_t^u and V_t^g are still described by (15a) and (15b), whereas $V_t^b = L$. At the early stage of the lending relationship, only the uninformed and informed-good types roll over maturing loans. By contrast, a bank that has learned that the project is bad chooses to liquidate. Assumption 1 guarantees that liquidation possesses a higher value than continuing the project. By continuity, liquidation still has a higher payoff if type b needs to wait for a long time (until t_g in this case) to refinance. As a result, zombie lending is suboptimal because t_g is far into the future: the firm could default or fail prematurely before it reaches the market financing stage. The equilibrium is socially efficient in this region. The result $t_b > 0$ implies that the bank cannot conduct zombie lending all the time. In this sense, zombie lending is self-limiting.

Boundary Conditions: The following two boundary conditions are needed to pin down $\{t_b, t_g\}$:

$$V_{t_b}^b = L \tag{16a}$$

$$\dot{V}_{t_g}^g = \dot{\hat{V}}_{t_g}^g = \left(D^g - D^b\right) \eta q_{t_g} \left(1 - q_{t_g}\right). \tag{16b}$$

Equation (16a) is the indifference condition for the bad type to liquidate at t_b, which is the standard value-matching condition in optimal stopping problems. In this case, rolling over brings the same payoff L, and thus by continuity and monotonicity, the entrepreneur prefers liquidating when $t < t_b$ and rolling over when $t > t_b$. The second condition, smooth pasting, comes from the No-Deals condition and the belief monotonicity refinement. In the Appendix we show that if this condition fails, type g will have strictly higher incentives to switch to market financing before t_g. Intuitively, because a bad project's present value falls below the liquidation value, the equilibrium decision of refinancing with the market must be one with pooling. Given the pooling structure in market refinancing, the smooth-pasting condition solves the optimal-stopping-time problem for the good types. The smooth-pasting condition picks the earliest t_g for the good entrepreneur to refinance with the market. With the boundary conditions, we can uniquely pin down $\{t_b, t_g\}$, as given by the following proposition.

PROPOSITION 2: *A* $\bar{\eta}$ *exists such that if* $\eta < \bar{\eta}$ *and* $V_0^u \geq \max\left\{L, \bar{V}_0^u\right\}$*, a unique monotone equilibrium exists in the absence of financial constraints and is characterized by thresholds* t_b *and* t_g*, where*

$$t_b = \frac{1}{\lambda+\eta}\left[\log\left(\frac{1-q_0}{q_0}\frac{\bar{q}}{1-\bar{q}}\right) - \eta(t_g - t_b)\right] \tag{17}$$

$$t_g - t_b = \frac{1}{r+\phi+\eta}\log\left(\frac{\bar{V}_{t_g}^b - PV_r^b}{L - PV_r^b}\right) \tag{18}$$

and $\bar{q}$ *solves*

$$\bar{q}^2 - \left(1 - \frac{r+\phi}{\eta}\right)\bar{q} + \frac{r+\phi}{\eta}\left(\frac{D^b}{D^g - D^b} - \frac{\delta+\phi}{r+\phi}\frac{D^g}{D^g - D^b}\right) = 0. \tag{19}$$

The condition $\eta < \bar{\eta}$ is not necessary but helps simplify the exposition. Intuitively, as η becomes sufficiently low, the No-Deals condition is always slack for the uninformed type after $t = 0$ so that they would never be interested in refinancing with only bad types.[16] The other condition, $V_0^u \geq \max\left\{L, \bar{V}_0^u\right\}$, requires that the uninformed type chooses bank financing at $t = 0$ – the continuation value exceeds both the value of immediate market financing and liquidation. In the Appendix, we provide a closed-form expression for V_0^u that allows us to write this condition in term of primitives.

Proposition 2 shows that the length of the zombie lending period (equation (18)) is sufficiently long to deter bad types from mimicking others at t_b: whereas $\bar{V}_{t_g}^b - PV_r^b$ captures the additional benefit of zombie lending until t_g, the denominator in the logarithm function $L - PV_r^b$ captures the relative benefit of liquidating the project at t_b. Equation (17) shows that the length of the efficient liquidation period t_b gets shorter as public news arrival becomes more likely (i.e., higher $\eta\,(t_g - t_b)$) during the zombie lending period. Intuitively, when public news is more likely to reveal the project's type, the project's reputation grows faster. Therefore, the length of the initial efficient liquidation stage, during which reputation grows without liquidation, is necessarily shorter.

Our core mechanism shares similarities with Hwang (2018). On the specific results of equilibrium in the last two regions, the difference is a matter of equilibrium selection, which lies between pure strategies and mixed strategies. In the absence of external news ($\eta = 0$), our pure-strategy equilibrium is payoff-equivalent to the mixed-strategy equilibrium identified in Hwang (2018). In particular, in Hwang (2018), there is an expected delay in receiving a high offer, whereas in our paper the delay in receiving the high offer is deterministic. Moreover, the efficiency benchmark in our paper is different

[16] If η becomes very high, the average belief on the uninformed type increases quickly after $t = 0$ so that the No-Deals condition for type u may bind after $t = 0$ even if it holds at $t = 0$. In other words, the uninformed types' incentives to pool with bad types can be non monotonic or even increase over time. These cases are analyzed in the Appendix.

from existing papers on a dynamic lemons market. Comparison of Propositions 1 and 2 immediately highlights inefficiencies with the good and bad types: delay in market financing occurs for a good-type project, which is similar to the standard inefficiency in the dynamic lemons literature, while a bad project is no longer liquidated after t_b. Moreover, comparison of μ_{t_g} and μ_{FB} highlights an interesting source of inefficiency for the uninformed type: the uninformed type obtains market financing at t_g, which is too soon.[17]

COROLLARY 1: *Under the parametric conditions in Proposition 2* , $\mu_{t_g} < \mu_{FB}$.

Note that this last source of inefficiency is the opposite of the inefficiency in the dynamic lemons literature. The inefficiency in our model is not the existence of delay, but rather insufficient delay. The uninformed types give up the option value of information after t_g due to the option of market refinancing.

REMARK 4. We specify a pessimistic belief during $\left[t_b, t_g\right)$ that is off the equilibrium path: any entrepreneur who seeks market financing during this period will be treated as a bad one and hence will be unable to refinance with the market. As in other signaling models, multiple off-equilibrium beliefs exist that could sustain the equilibrium outcome. The pessimistic belief is one of them, and perhaps the one most commonly used. In Section IV.A, we consider an extension in which the lending relationships may break up exogenously, so some entrepreneurs always seek market financing on the equilibrium path, and hence specifying off-equilibrium beliefs is unnecessary. The structure of the equilibrium is similar, and we show that the equilibrium outcome converges to the one in our model when the probability of the exogenous breakup goes to zero. We can show that the market belief in the limit is the one that makes the bad type indifferent between rolling over bad loans and immediately financing with the market, and no discontinuity exists in beliefs at t_g. In other words, the refinement selects an off-equilibrium belief that is continuous in time. That said, throughout the paper we continue to use the pessimistic off-equilibrium belief because it is more convenient and commonly used in the literature.

B. *Equilibrium under the Financial Constraint $y_t \leq c$*

Our benchmark case applies to a scenario in which the Coase theorem holds, so that frictionless bargaining and negotiation will lead to the efficient allocation between the entrepreneur and the bank. Therefore, at each rollover date, a loan will be rolled over if the joint surplus is above the liquidation value L. In this subsection, we formally analyze the model with the financial constraint $y_t \leq c$. Clearly, the Coase theorem no longer applies and hence we need to study the incentives of the bank and the entrepreneur

[17] Note that under $\eta = 0$, $\mu_{t_g} = q_0$ so that the result holds trivially. Under continuity, the corollary holds for η sufficiently small.

separately.[18]

The HJBs for the value function $\{V_t^i,\ i \in \{u, g, b\}\}$ remain unchanged from those in Section II.A. Again, we can use two thresholds $\{t_b, t_g\}$ to characterize the equilibrium solutions. Let us now turn to the boundary conditions. First, the smooth-pasting condition continues to hold, because it selects the equilibrium in which a good-type entrepreneur chooses to refinance with the market as early as possible. Note that the smooth-pasting condition pins down $\bar{q}$, implying that the average quality financed by the market remains unchanged under the financial constraint. The second boundary condition, value matching at t_b, is different. In particular, because the entrepreneur is financially constrained and cannot repay its loan before t_g, the bank has the right to liquidate the project. It chooses to roll over the loan only if its continuation value lies above L. As a result, the value-matching condition at t_b becomes

$$B_{t_b}^b = L \tag{20}$$

instead of $V_{t_b}^b = L$.

PROPOSITION 3: *If $B_0^u \geq L$, then under the financial constraint $y_t \leq c$ and the same parametric conditions in Proposition 2, the equilibrium is characterized by the two thresholds $\{t_b, t_g\}$,*

$$t_b = \frac{1}{\lambda + \eta}\left[\log\left(\frac{1-q_0}{q_0}\frac{\bar{q}}{1-\bar{q}}\right) - \eta(t_g - t_b)\right] \tag{21a}$$

$$t_g - t_b = \frac{1}{r + \phi + \eta}\log\left(\frac{F - \frac{c+\phi\theta F}{r+\phi+\eta}}{L - \frac{c+\phi\theta F}{r+\phi+\eta}}\right), \tag{21b}$$

where $\bar{q}$ remains unchanged from Proposition 2.

Comparison of Propositions 2 and 3 shows that the financial constraint $y_t \leq c$ mitigates the inefficiency from zombie lending.

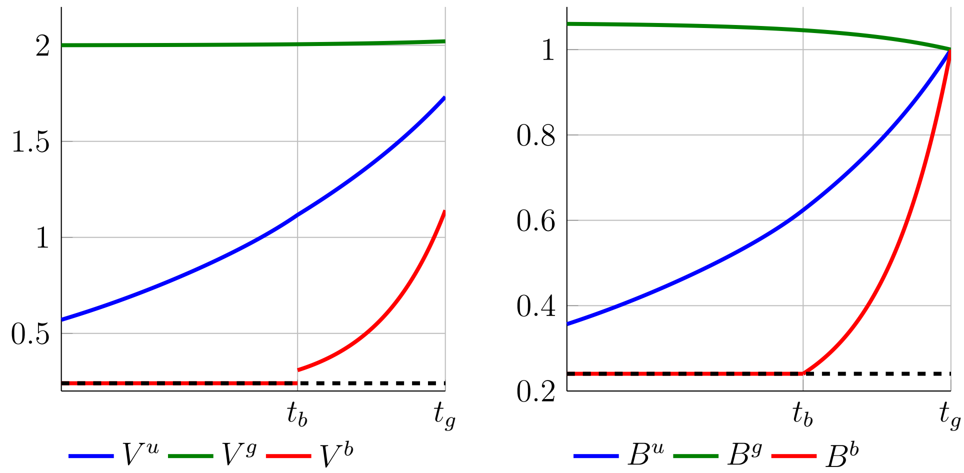
COROLLARY 2: *The length of the zombie lending period $t_g - t_b$ becomes shorter under the financial constraint $y_t \leq c$, while t_b becomes larger, so that the period of efficient liquidation becomes longer.*

Intuitively, the financial constraint $y_t \leq c$ limits the size of repayments that the entrepreneur is able to make to the bank. Therefore, the constraint allows the bank's continuation value to fall below the liquidation value L, even though the joint surplus is still above L. As a result, the bad project is liquidated more often, compared to the case without the financial constraint. Consequently, the length of the zombie lending period $t_g - t_b$ becomes shorter. This result highlights the role of financial constraints and

[18] Another financial constraint exists whereby the bond price at t_g must be at least F, implying $\bar{q} \geq \frac{F-D^b}{D^g-D^b}$. This condition turns out to always be slack, so we focus on the constraint $y_t \leq c$.

their interaction with asymmetric information among different types of lenders. Whereas most existing empirical research on zombie lending emphasizes the effect of a financially constrained bank, our theory offers a new testable implication on the effect of financially constrained firms in the lending relationship. In particular, our results imply that as firms become more financially constrained, zombie lending could be mitigated.

Numerical Example: Figure 2 plots the value function of all three types: whereas the left panel shows the joint valuations of the entrepreneur and the bank, the right one only shows those of the bank. In this example, $t_b = 1.2921$ and $t_g = 2.1006$. In the figure, the green, blue, and red lines represent the value functions of the informed-good, the uninformed, and the informed-bad types, respectively. The dashed horizontal line marks the levels of L. Before t reaches t_b, the bad type's value function stays at L, and all of the continuation value accrues to the bank. Note that at t_b, the bad-type entrepreneur's value function experiences a discontinuous jump, whereas no such jump occurs in the bank's value function. This contrast is due to the financial constraint $y_t \leq c$. Indeed, both value functions are smooth without this constraint.



Panel A. Path of joint continuation value

Panel B. Path of bank's continuation value

Figure 2. Value functions. This figure plots the value functions $V^i, B^i, i \in u, g, b$ with the following parameter values: $r = 0.1$, $\delta = 0.02$, $m \to 0$, $F = 1$, $\phi = 1.5$, $R = 2$, $c = 0.2$, $\theta = 0.1$, $L = 1.25 \times NPV_r^b$, $\lambda = 2$, $\eta = 1$, and $q_0 = 0.2$.

Under Assumption 3, an informed-good bank can in principle charge an interest rate that is above the cost of capital r, even though the loan will always be repaid. The private information therefore enables the bank to earn some rents. As illustrated in the right panel of Figure 2, these rents, or equivalently, the informed-good bank's value function (green line of the right panel), decrease over time. This pattern illustrates the dynamics of the bank's ability to extract rents in a lending relationship. As time approaches t_g and the termination of the lending relationship nears, this ability to extract excessive rents from a good-type entrepreneur becomes more limited. This result highlights a distinction of our paper from the literature on loan sales and securitization.[19] In loan sales and securitization, banks with good loans choose to retain a larger share of the loans (or more junior tranches) for a longer period of time to signal the loans' quality. The benefit is that by doing so, they receive more proceeds by selling the loans at higher prices. In our context, however, a good bank has the opposite incentive. It does not want to signal that its borrower is good. Instead, the good bank prefers to extract surplus in the lending relationship as long as possible. Once the borrower refinances with the market, the bank no longer receives any extra proceeds above the full repayment of the loan. Therefore, a good-type bank prefers to keep its borrower in the lending relationship, as opposed to selling or securitizing the loan.

C. No Premature Failure

In this subsection, we study a special case of our model in which no premature failures occur, that is, $\eta \equiv 0$. As a result, μ_t, the (naive) belief update from no premature failure, will always stay at q_0. Proposition 4 shows the results, in which we obtain simple and closed-form solutions for $\bar{q}$ and $t_g - t_b$.

PROPOSITION 4: *If $\eta = 0$ so that no premature failure occurs, the equilibrium is characterized by thresholds $\{\bar{q}, t_b, t_g\}$, where*

$$\bar{q} = \frac{\frac{\delta+\phi}{r+\phi} D^g - D^b}{D^g - D^b} \tag{22}$$

$$t_g - t_b = \frac{1}{r+\phi} \log \left(\frac{F - \frac{c+\phi\theta F}{r+\phi}}{L - \frac{c+\phi\theta F}{r+\phi}} \right). \tag{23}$$

Compared to the case without premature failure ($\eta = 0$), the case with premature failure ($\eta > 0$) has a higher $\bar{q}$ and lower $t_g - t_b$.

Equation (22) is a standard result in the dynamic lemons literature,[20] that is obtained by solving $q_t D^g + (1 - q_t) D^b = \frac{\delta+\phi}{r+\phi} D^g$. The left-hand side, $q_t D^g + (1 - q_t) D^b$, captures the competitive price of the bond, whereas $\frac{\delta+\phi}{r+\phi} D^g$, captures the value of a good-type loan. Therefore, $\bar{q}$ is the minimum quality q such that the value of the good-type debt to the bank is equal to the market's willingness to pay for the debt of an average entrepreneur. The existence of premature failure ($\eta > 0$) reduces D^b and therefore increases $\bar{q}$. Moreover, the presence of premature failure renders zombie lending by bad types more costly, because the project could fail during this period. As a result, the period of zombie lending

[19] In practice, 60% of the loans are first sold within one month of loan origination and nearly 90% are sold within one year (Drucker and Puri (2009)). As Gande and Saunders (2012) argue, a special role of banks is to create an active secondary loan market while still producing information.

[20] See Lemma 3 of Hwang (2018), for example.

gets shorter. Proposition 4 implies that for firms with more transparent governance and accounting systems, the concern for zombie lending is mitigated.

Our next corollary provides interesting comparative static results on the amount of zombie lending and credit quality with respect to primitive variables.

COROLLARY 3: *In the case of* $\eta = 0$, $\bar{q}$ *increases with* δ, *decreases with* r *and* θ, *and is unaffected by either* λ *or* L. *Moreover,* $t_g - t_b$ *decreases with* r, L, *and* θ, *and is unaffected by* δ *or* λ.

We offer some explanations for the results on r and δ. Note that the role of the zombie lending period is to discourage bad types from mimicking other types at $t = t_b$, as is clearly seen in (23): whereas $F - \frac{c+\phi\theta F}{r+\phi}$ captures the additional benefit of zombie lending until t_g, the denominator in the logarithm function $L - \frac{c+\phi\theta F}{r+\phi}$ captures the relative benefit of liquidating the project at t_b.

Intuitively, lower δ is associated with cheaper market financing. Therefore, $\bar{q}$, the average quality of borrowers that are eventually financed by the market, decreases. By contrast, if the cost of bank financing r becomes cheaper, credit quality $\bar{q}$ increases. Intuitively, if the bank's cost of capital becomes lower, gains from trade with the market are lower, so a good type only refinances with the market if the average quality becomes even higher.[21]

C.1. Initial Borrowing

Given that no asymmetric information exists at $t = 0$, and no bankruptcy cost exists, the entrepreneur would like to borrow as much as possible at the initial date. Therefore, without loss of generality, we can assume that the loan takes the maximum pledgeable income F, in which case the entrepreneur is able to raise at most B_0^u initially. If the entrepreneur needs to invest I at $t = 0$, the project can only be initiated if $B_0^u \geq I$. Proposition 5 describes the closed-form expression of B_0^u.

PROPOSITION 5: *In the case of* $\eta = 0$, *the entrepreneur's maximum borrowing amount at* $t = 0$ *is*

$$B_0^u = q_0 \left[\frac{c + \phi F}{r + \phi} + e^{-(r+\phi)t_b} \left(B_{t_b}^g - \frac{c + \phi F}{r + \phi} \right) \right] + (1 - q_0) \left[\frac{c + \phi\theta F + \lambda L}{r + \phi + \lambda} + e^{-(r+\phi+\lambda)t_b} \left(L - \frac{c + \phi\theta F}{r + \phi + \lambda} \right) \right], \quad (24)$$

[21] Obviously, if r becomes even lower than δ, the entrepreneur will never refinance with the market, and no zombie lending period exists.

where

$$B_{t_b}^g = \frac{c+\phi F}{r+\phi} + e^{-(r+\phi)(t_g-t_b)}\left(F - \frac{c+\phi F}{r+\phi}\right). \tag{25}$$

Intuitively, B_0^u in (24) has two components. With probability q_0, the project is good, in which case the bank is able to receive payments $\frac{c+\phi F}{r+\phi}$ until t_g, after which it is fully repaid. With probability $1-q_0$, the project turns out bad, and the bank has the option to liquidate it if the bad private news arrives before t_b.

An increase in the cost of bank financing r may increase or decrease B_0^u. On the one hand, all of the payments (interim and final repayments) are more heavily discounted when r increases. On the other hand, both $\bar{q}$ and $t_g - t_b$ become lower because the incentive to conduct zombie lending is lower. As a result, the entrepreneur is able to refinance with the market (in which case the bank is fully repaid) earlier. The overall effect thus depends on the relative magnitude of these two effects.

An increase in δ may also increase or decrease the initial borrowing amount B_0^u. When market financing becomes more expensive, $\bar{q}$ increases, as do t_b and t_g. However, the effect of δ on B_0^u includes two counterveiling effects. First, if the project turns out to be good, the bank is able to extract excessive rents for a longer period of time, which increases the amount that it is willing to lend up front. Second, for the fixed payments, the bank needs to wait longer to be fully repaid, which decreases the amount that it is willing to lend up front. In the proof in the Appendix, we offer details on conditions that characterize the monotonicity and we show that, in general, an increase in δ first decreases and then increases B_0^u.

D. General Maturity

Our analysis so far focuses on the case of instantly maturing loans ($m \to 0$). In this subsection, we describe the results for the general case in which loans have expected maturity m. We show that all of our previous results continue to go through.[22] Moreover, we show how t_b, $t_g - t_b$, and $\bar{q}$ vary with loan maturity m. For simplicity, we focus on the case without premature failure, by taking $\eta = 0$.

When loans mature gradually, bad projects are also liquidated gradually as their loans mature during $[0, t_b]$. In Internet Appendix III, Lemma IA0 describes the evolution of public beliefs without liquidation.[23] Moreover, we can generalize the HJB equation

[22]Note that the market financing region under general maturity $m > 0$ is $[t_g, \infty)$, depending on when the existing bank loan matures after t_g.

[23]The Internet Appendix is available in the online version of the article on the Journal of Finance website

systems into

$$(r+\phi)\,V_t^u = \dot{V}_t^u + c + \phi\left[q_0 + (1-q_0)\,\theta\right]R \qquad (26a)$$
$$+\lambda\left[q_0 V_t^g + (1-q_0)\,V_t^b - V_t^u\right] + \frac{1}{m}\mathcal{R}(V_t^u, \bar{V}_t^u)$$
$$(r+\phi)\,V_t^g = \dot{V}_t^g + c + \phi R + \frac{1}{m}\mathcal{R}(V_t^g, \bar{V}_t^g) \qquad (26b)$$
$$(r+\phi)\,V_t^b = \dot{V}_t^b + c + \phi\theta R + \frac{1}{m}\mathcal{R}(V_t^b, \bar{V}_t^b), \qquad (26c)$$

where

$$\mathcal{R}(V_t^i, \bar{V}_t^i) \equiv \max\left\{0, \bar{V}_t^i - V_t^i, L - V_t^i\right\}. \qquad (27)$$

Note that the equation system is identical to (15a) to (15c), except for the last terms, which account for the loan maturing. In this case, the bank and the entrepreneur choose between rolling over the debt (zero in equation (27)), replacing the loan with the market bond ($\bar{V}_t^i - V_t^i$ in (27)), and liquidating the project ($L - V_t^i$ in (27)). Note that under general maturity, the entrepreneur does not get to refinance immediately after t reaches t_g. Therefore, the expressions for $\bar{V}_t^i$ are different from (5), and we supplement them in Internet Appendix III.

The boundary conditions are unchanged. Again, we characterize the equilibrium in three regions.

PROPOSITION 6: *If the loan has general maturity m, a unique m^* exists such that the equilibrium has three stages if $m < m^*$. The liquidation threshold is given by*

$$t_b = \min\left\{t > 0 : \frac{q_0\left(1 - q_0 + q_0 e^{\lambda t}\right)^{\frac{1}{\lambda}-1} e^{\frac{1}{m}t}}{1 + \frac{1}{m}\int_0^t \left(1 - q_0 + q_0 e^{\lambda s}\right)^{\frac{1}{\lambda}} e^{\left(\frac{1}{m}-\lambda\right)s} ds} = \bar{q}\right\}, \qquad (28a)$$

and $\bar{q}$ follows (22).

1. *Without the financial constraint $y_t \le c$,*

$$t_g - t_b = \frac{1}{r+\phi}\log\left(\frac{V_{t_g}^b - \frac{c+\phi\theta R}{r+\phi}}{L - \frac{c+\phi\theta R}{r+\phi}}\right), \qquad (29)$$

where

$$V_{t_g}^b = \frac{c+\phi R}{r+\phi} - \frac{\phi R\,(1-\theta) + \frac{1}{m}\frac{\phi(R-F)(1-\theta)}{r+\phi}}{r+\phi+\frac{1}{m}}. \qquad (30)$$

2. *Under the financial constraint $y_t \leq c$,*

$$t_g - t_b = \frac{1}{r+\phi} \log \left(\frac{\frac{r+\phi}{r+\phi+1/m}(c + \phi\theta F + \frac{1}{m}F) - (c + \phi\theta F)}{(r+\phi)L - (c+\phi\theta F)} \right). \tag{31}$$

A simple comparison with the results in Section II.C shows that when m increases, $\bar{q}$ is unchanged and t_b increases, whereas $t_g - t_b$ decreases.[24] Intuitively, $\bar{q}$ is determined as the lowest average quality at which a good-type entrepreneur is willing to refinance with the market. In this case, the market financing condition is such that good types receive the identical payoff as staying with the bank and refinancing with the market. Thus, $\bar{q}$ does not vary with the maturity of the loan.[25] It takes longer for the average quality to reach $\bar{q}$ when the maturity m increases, because bad projects are liquidated less frequently. Therefore, t_b increases. Finally, after t reaches t_g, bad types take longer to refinance with the market, and therefore $V^b_{t_g}$ decreases with m. Therefore, a shorter period $t_g - t_b$ could still deter bad types from mimicking at t_b.

When $m > m^*$ so that the maturity of the loan becomes sufficiently long, the equilibrium is characterized by one single time cutoff t_{bg}. From t to t_{bg}, bad projects are liquidated, whereas market financing occurs right after t_{bg}. The boundary condition is captured by the value-matching condition $V^b_{t_{bg}} = L$.[26] Intuitively, the zombie lending period is necessary to incentivize the bad types to liquidate early. When the maturity of the loan becomes long enough, even if the market financing stage has arrived, the bad types still need to wait until the loan matures to refinance with the market. For a higher m, the expected length of this period increases, so the project is more likely to mature before the next rollover date.

III. Endogenous Learning

Our analysis so far assumes that learning and (private) news arrival is an exogenous process that occurs as long as the entrepreneur has an outstanding bank loan. In this section, we analyze the model in which learning is endogenously chosen by the bank as a costly decision. We show that the equilibrium structure is still captured by thresholds $\{t_b, t_g\}$. An interesting result is that even if the cost of learning is small, the bank will stop producing information before t reaches t_b. Note that this result holds even if the bank has *all* of the bargaining power in the lending relationship. Therefore, our analysis highlights a new type of hold-up problem in relationship banking: the bank undersupplies effort in producing valuable information.

[24] The threshold t_g may increase or decrease, depending on the magnitude of λ and $r + \phi$.

[25] Mathematically, the smooth-pasting condition, $\dot{V}^g_{t_g} = 0$, leads to this result.

[26] The smooth-pasting condition no longer holds. In general, $\frac{dV^g_{t_{bg}}}{dt} \geq 0$.

Throughout this section, we assume that $\eta = 0$ so no premature failure occurs. We present the results with the financial constraint $y_t \leq c$; the case without the constraint yields qualitatively similar results. The structure of the model is unchanged from Section I, except that banks must learn the private news by choosing a rate $a_t \in [0,1]$. Given a_t, private news arrives at Poisson rate λa_t, and our previous analysis corresponds to the case in which $a_t \equiv 1$. Clearly, a higher rate leads to earlier arrival of private news in expectation. Meanwhile, learning incurs a flow cost ψa_t so that a higher rate is also more costly to the bank. Heuristically, within a short period $[t, t+dt]$, the learning benefit is $\lambda a_t \left[q_0 B_t^g + (1-q_0) B_t^b - B_t^u\right] dt$: with probability $\lambda a_t dt$, private news arrives, at which time the bank receives continuation payoff B_t^g with probability q_0 and B_t^b with probability $1 - q_0$. The cost of learning is approximately $\psi a_t dt$ during the period. Given the linear structure, the bank's learning decision follows a bang-bang structure. Specifically, it chooses maximum learning ($a_t = 1$) if and only if

$$\lambda \left[q_0 B_t^g + (1-q_0) B_t^b - B_t^u\right] \geq \psi. \tag{32}$$

Otherwise, it chooses not to learn at all and $a_t = 0$.

PROPOSITION 7: *1. If $\frac{\psi}{\lambda} < \frac{\frac{1}{m}(1-q_0)}{r+\phi+\frac{1}{m}}\left(L - \frac{c+\phi\theta F}{r+\phi}\right)$, an equilibrium characterized by $\{t_a, t_b, t_g\}$ and $t_a < t_b < t_g$ exists. The bank learns if and only if $t < t_a$.*

2. Otherwise, the bank never learns and the entrepreneur never borrows from the bank.

We offer some intuition behind Proposition 7. If the bad project no longer gets liquidated, the value of an uninformed bank is a linear combination of an informed-good one and an informed-bad one, that is, $B_t^u = q_0 B_t^g + (1-q_0) B_t^b$, which is the case after t reaches t_b. As a result, after t reaches t_b, the benefit of learning is zero, implying that in any equilibrium, banks may only learn for $t \leq t_b$. During $[0, t_b)$, when the bad projects still get liquidated, the value of becoming informed is positive because liquidation avoids the expected loss generated from a bad project (see Figure 3 for a graphical illustration.), that is, $q_0 B_t^g + (1-q_0) L - B_t^u > 0$. In this case, information is valuable. The proposition above shows that if the cost of learning is sufficiently low, then the bank learns until t_a. If the cost is relatively high, however, the bank will never learn and thus entrepreneurs will never choose bank financing.

The thresholds $\{t_a, t_b, t_g\}$ are given by the solution to the system of equations (IA7) in the Internet Appendix. Corollary 4 offers the expression in the limiting case of zero maturity.

COROLLARY 4: *As $m \to 0$, the thresholds in Proposition 7 converge to*

$$t_a = \frac{1}{\lambda}\left[\log\left(\frac{\bar{q}}{1-\bar{q}}\right) - \log\left(\frac{q_0}{1-q_0}\right)\right]$$

$$t_b = t_a - \frac{1}{r+\phi}\log\left(1 - \frac{\psi/\lambda}{(1-q_0)\left(L - \frac{c+\phi\theta F}{r+\phi}\right)}\right)$$

$$t_g = t_a + \frac{1}{r+\phi}\log\left(\frac{F - \frac{c+\phi\theta F}{r+\phi}}{L - \frac{c+\phi\theta F}{r+\phi}}\right),$$

and t_b is higher compared to the case with exogenous learning as in Proposition 6.

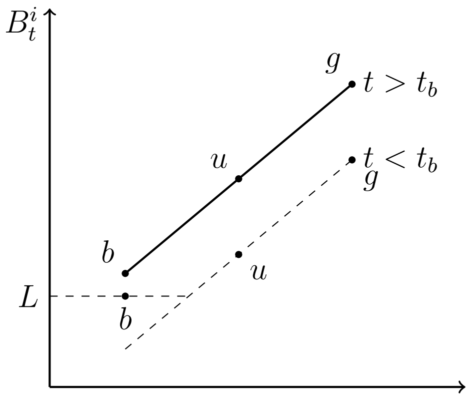


Figure 3. Graphical illustration of learning benefits.

Note that for $t \in [t_a, t_b]$, the bank chooses not to learn in equilibrium. Off the equilibrium path, if the bank chose to learn, it would liquidate the project upon bad news. The reason that t_b needs to be strictly higher than t_a is to generate positive benefits from learning. When the learning cost $\psi \to 0$, t_b converges to t_a.

Under endogenous learning, $\bar{q}$ and $t_g - t_b$ are unchanged whereas t_b increases. The reason is that $\bar{q}$ is determined by the good type's indifference condition between bank financing and market financing, whereas $t_g - t_b$ is the length of the zombie lending period that is just sufficient to deter bad types from mimicking others at t_b. Because both $\bar{q}$ and $t_g - t_b$ are determined by types that are already informed, they are unaffected when producing information becomes costly and endogenous. Finally, because less information is produced when learning becomes costly, bad types are liquidated less often and the average quality q_0 takes longer to reach $\bar{q}$, resulting in a higher t_b.

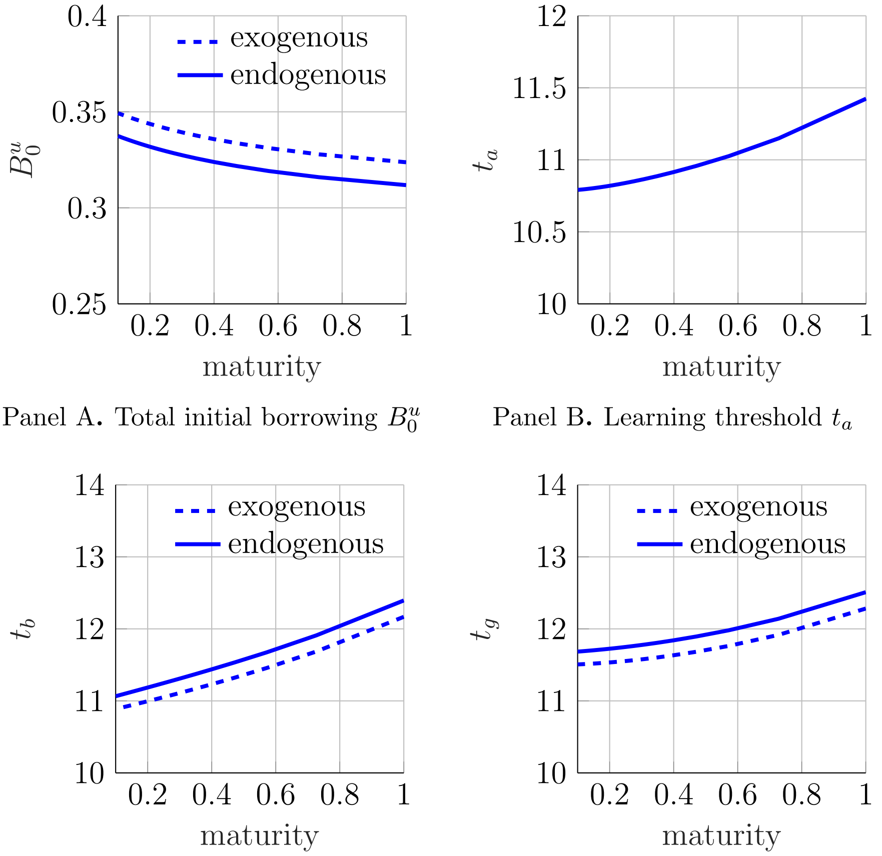
Proposition 7 highlights a new type of hold-up problem in relationship lending that only emerges in the dynamic setup. Rajan (1992) shows that in a lending relationship, the entrepreneur has an incentive to underinvest effort due to the prospect of renegotiation following private news. Our paper shows that the relationship bank will also underinvest effort in producing information even if the bank has *all* of the bargaining power and the cost of producing information is infinitesimal (but still positive). The reason is that the prospect of future market refinancing prevents the bank from capturing all of the surplus generated from information production, even though it has all of the bargaining power. Knowing so, the bank undersupplies effort in producing information.[27]

Numerical Example: Under the same set of parameters as in Section II.B (except for $\eta = 0$), with the additional parameter that $\psi = 0.06$ and $m = 1$, we get $t_a = 3.8350$, $t_b = 4.7861$, and $t_g = 5.1352$.

Figure 4 illustrates the effect of loan maturity on equilibrium results. The dashed lines plot the same results under Proposition 6, where private news arrives exogenously. The differences between the solid and the dashed lines therefore capture the contribution of endogenous learning. In general, two effects arise when the length of loan maturity increases. First, a longer maturity reduces the option value of new information, because the bank must wait until the rollover date to act on new information. As a result, the incentive to produce information should be lower. Second, a longer maturity increases the risk that banks face by rolling over bad loans, which increases in turn banks' incentives to learn. In our numerical exercise, the second effect dominates so that t_a, the boundary at which the bank stops learning, increases with loan maturity. Therefore, B_0^u, the amount of initial borrowing, decreases to compensate for the increased learning cost. The bottom two panels show that as a result, both t_b and t_g increase, whereas the difference, $t_g - t_b$ is unchanged.[28]

[27] Diamond, Hu, and Rajan (2020) and Diamond, Hu, and Rajan (2019) have a similar flavor, showing that high prospective liquidity (akin to the availability of market financing here) results in reduced corporate governance and bank monitoring.

[28] When maturity becomes even longer, the effect becomes non monotonic. In the extreme case in which the loan never matures, private news is useless and $t_a = 0$. This latter pattern is captured by the second case of Proposition 7, in which bank financing is not used in equilibrium (equivalently, ψ gets very high).



Panel A. Total initial borrowing B_0^u

Panel B. Learning threshold t_a

Panel C. Liquidation threshold t_b

Panel D. Zombie lending threshold t_g

Figure 4. Comparative statics with endogenous learning. This figure plots the value function with the following parameter values: $r = 0.1$, $\delta = 0.05$, $m = 1$, $F = 1$, $\phi = 1.5$, $R = 2$, $c = 0.2$, $\theta = 0.1$, $L = 1.5 \times NPV_r^b$, $\lambda = 0.5$, and $q_0 = 0.1$, $\psi = 0.025$.

IV. Extension and Empirical Relevance

A. *Lending-Relationship Breakups*

In practice, lending relationships may break up for reasons independent of the underlying project's quality. For instance, the relationship may have to be terminated if the bank experiences shocks that dry up its capital or funding. In this subsection, we modify the model setup by assuming that at rate $\chi > 0$, the lending relationship breaks up, at which point the entrepreneur is forced to refinance with the market or the project is liquidated immediately. For simplicity, we focus on the case without either the premature failure ($\eta = 0$) or the financial constraint $y_t \leq c$.

Note that with some probability, types u and g refinance with the market. As a result, the bond price always exists on the equilibrium path. An equilibrium is therefore defined as in Definition 1 without the refinement of No-Deals and belief monotonicity.

PROPOSITION 8: *A $\underline{q}$ exists such that if $q_0 < \underline{q}$, an equilibrium characterized by thresholds $\{t_\ell, t_b, t_g\}$ exists. The decisions of the good and uninformed types are identical to those in Proposition 4.*

1. *If $t \in [0, t_\ell]$, bad types liquidate their projects upon learning.*

2. *If $t \in [t_\ell, t_b]$, bad types liquidate their projects with probability $\ell_t \in (0,1)$ upon learning. With probability $1 - \ell_t$, bad types refinance with the market.*

3. *If $t \in \big[t_b, t_g\big)$, bad types refinance with the market at some rate $\alpha_t > 0$.*

4. *If $t = t_g$, bad types refinance with the market immediately.*

Figure 5 illustrates the equilibrium strategies in Proposition 8, which has the same qualitative features as that in Section II.A. However, the equilibrium in this modified game necessarily involves bad types using mixed strategies. When $t \in [t_\ell, t_b]$, the bad types are indifferent between liquidating and refinancing with the market. In equilibrium, liquidating happens with probability ℓ_t, so that the average quality of firms that refinance with the market lies strictly above q_0 on the equilibrium path. Panel 5A plots the probability of liquidation ℓ_t. Note that ℓ_t decreases with t during $[t_\ell, t_b]$, so that q_{t+}, the quality of the project conditional on market refinancing, as well as the bond price stay constant.

When $t \in \big[t_b, t_g\big)$, bad types play a mixed strategy between bank financing and market financing, implying that some degree of zombie lending exists. Note that bad types cannot always remain in the lending relationship, because the equilibrium bond price will be too high. Instead, they voluntarily refinance with the market at a strictly positive rate even without the exogenous breakup.[29] Panel 5B plots $\gamma_t = \alpha_t \pi_t^b$, the flow rate of bad types that voluntarily seek market financing without the breakup on $\big[t_b, t_g\big)$.[30] As time increases, γ_t decreases.

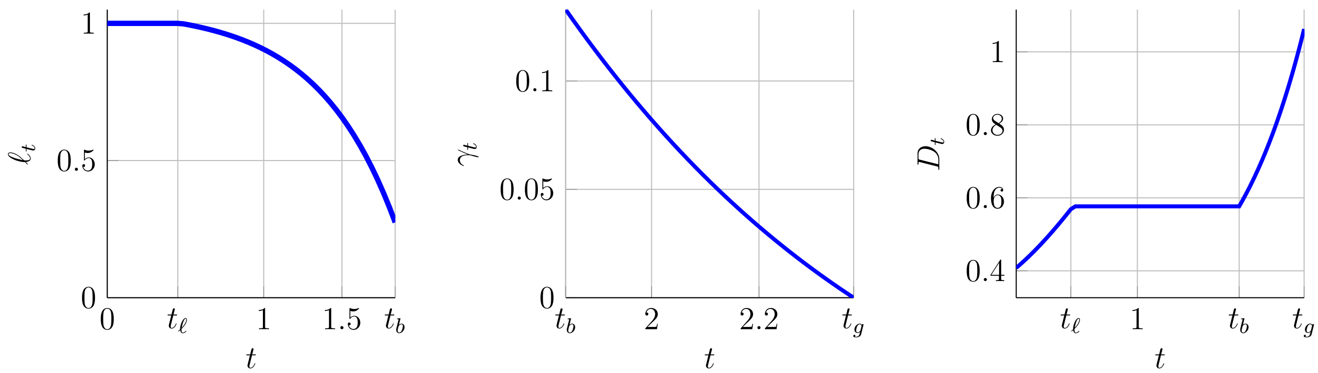
Panel 5C plots the bond price for borrowers who seek market financing between $[0, t_g]$. The price pattern is consistent with bad types using mixed strategies in equilibrium. Between $t = 0$ and t_ℓ, the price increases as bad types liquidate their projects. The price becomes a constant between t_ℓ and t_b, so that a bad type is indifferent between liquidation and market refinancing. After t reaches t_b, the bond price needs to increase to make bad types indifferent between bank and immediate market financing.

The equilibrium of this modified game converges to the one in Section II.A as $\chi \to 0$.[31] The average quality of the entrepreneurs who seek financing between t_b and t_g, however,

[29] Note that they cannot refinance with an atomistic probability, because the bond price will then fall to D^b.

[30] The total flow of bad types is $\chi \pi_t^b + \gamma_t$.

[31] This limit can be interpreted as a refinement of the equilibrium in the spirit of trembling-hand-perfect equilibria (Fudenberg and Tirole (1991)).



Panel A. Probability of liquidation, ℓ_t

Panel B. Voluntary flow of bad projects into market, $\gamma_t = \alpha_t \pi_t^b$

Panel C. Bond price D_t

Figure 5. Equilibrium with exogenous breakups. This figure plots the equilibrium strategies ℓ_t, γ_t and bond prices D_t for the following parameter values: $r = 0.1$, $\delta = 0.02$, $m = 10$, $F = 1$, $\phi = 1.5$, $R = 2$, $c = 0.2$, $\theta = 0.1$, $L = 1.2 \times NPV_r^b$, $\lambda = 2$, $q_0 = 0.2$ and $\chi = 0.1$. The equilibrium thresholds are $t_\ell = 0.45$, $t_b = 1.84$, and $t_g = 2.37$.

does not converge to zero. Instead, it converges to q_{t+}. In a game with $\chi \equiv 0$, if we impose the off-equilibrium belief $q_t = q_{t+}$ $\forall t \in [t_b, t_g)$, only the bad entrepreneurs will choose to voluntarily refinance with the market. Therefore, the game with $\chi \to 0$ can serve as a microfoundation to justify the discontinuity in beliefs in the game with $\eta = 0$ and $\chi = 0$.

Proposition 8 implies that conditional on market refinancing, the average quality of firms increases with the length of the lending relationship. This result is consistent with the negative-announcement effect of debt initial public offering, as we explain next.

B. *Empirical Relevance*

In this subsection, we provide consistent empirical evidence and derive the model's testable implications.

B.1. Dynamic Information Production and Liquidation

Our paper builds on the key assumption that a relationship bank acquires superior information not upon its first contact with a borrower, but through repeated interactions during the prolonged relationship. This assumption is motivated by evidence in James (1987) and especially Lummer and McConnell (1989), who find no abnormal returns to the announcement of new loans but strong abnormal returns associated with loan renewals.[32] Moreover, renewals with favorable (unfavorable) terms have positive (negative) abnormal

[32] Slovin, Johnson, and Glascock (1992), Best and Zhang (1993), and Billett, Flannery, and Garfinkel (1995) document positive and significant price reactions to both loan initiation and renewal announcements.

returns, suggesting the importance of asymmetric information.[33] Our result on zombie lending implies that as the lending relationship continues, renewals should gradually contain more favorable terms but the positive abnormal returns will shrink. The result on efficient liquidation predicts that the age distribution of liquidated loans is left-skewed.[34]

B.2. Zombie Lending

A central result of our model is that relationship banks conduct zombie lending to cover negative private information, so that they can offload these loans to other lenders in the near future. These other lenders can be nonbank institutions or other banks with funding advantages. The most direct evidence for this channel is presented by Gande et al. (1997), who study debt underwriting by commercial banks and investment houses. They show that when debt securities are issued for purposes other than repaying existing bank debt, the yield spreads are reduced by 42 bps if underwritten by commercial banks. Interestingly, when the stated purpose is to refinance existing bank debt, there is no statistical significance between yield spreads on debt issues underwritten by commercial banks and investment houses.

Some anecdotal evidence also suggests the channel highlighted in our paper. An example is Horizon Bank in Washington, which failed in 2010. According to its Material Loss Review, Horizon Bank frequently renewed, extended, or modified its large relationship loans without taking adequate steps to ensure that the borrower had the capacity to repay the loan. Loan files often cited refinancing as the sole exit strategy in the event of problems (pp. 7 and 9 in FDIC (2010)). In practice, many troubled loans are eventually refinanced by others, which in some cases even leads to the failure of the banks that buy these loans. An example is FirstCity Bank of Stockbridge in Georgia, which failed in 2009. According to its Material Loss Review, the bank adopted inadequate loan policies, and no analysis was made of possible liquidation values in the event a project did not perform (p. 6 of FDIC (2009)). Gordon Bank in Georgia, which purchased many loan participations from FirstCity without performing adequate due diligence,[35] subsequently failed in 2011 (FDIC (2011)). Relatedly, Giannetti, Liberti, and Sturgess (2017) show

[33] More recently, Botsch and Vanasco (2019) provide evidence that loan contract terms change over time as banks learn about borrowers. In particular, relationship lending benefits are heterogeneous, with higher-quality borrowers experiencing declining prices and lower-quality borrowers experiencing increasing prices and declining credit supply. This evidence is consistent with our key assumption that a relationship bank acquires superior information through repeated transactions with the borrower.

[34] In practice, bank loans are often secured. It is widely believed that lenders obtain more bargaining power upon seizing the asset and push for liquidation. See Benmelech, Kumar, and Rajan (2020) and the citations therein.

[35] Loan participation is defined as the transfer of an undivided interest in all or part of the principle amount of a loan from a seller, known as the "lead," to a buyer, known as the "participant," without recourse to the lead, pursuant to an agreement between the lead and the participant. "Without recourse" means that loan participation is not subject to any agreement that requires the lead to repurchase the participants interest or to otherwise compensate the participant upon the borrower's default on the underlying loan.

that for low-quality borrowers with multiple lenders, a relationship bank upgrades its private credit rating about the borrower to avoid other lenders cutting credit and thus impairing the borrower's ability to repay loans.[36]

B.3. Market Financing

Our model's market financing stage can be interpreted in various ways. The most direct interpretation is debt initial public offering. Datta, Iskandar-Datta, and Patel (2000) show that an initial public debt offering has a negative stock price effect, with the effect stronger for younger firms. Our model with exogenous breakup is consistent with this pattern (Panel 5C of Figure 5). Early on, only bad projects are voluntarily refinanced with the market without lending-relationship breakups. Only during later stages do the good and uninformed types start to voluntarily refinance with the market as well. A complementary hypothesis that remains untested is that the announcement effect of loan renewals preceding public debt issuance (or loan sales) should be small or even zero. An alternative interpretation of market financing is a credit rating upgrade from speculative to investment bucket, which, as Rauh and Sufi (2010) show, leads to firms shifting heavily away from bank loans to bonds. Our model predicts that relationship banks are more likely to conduct zombie lending before debt initial public offerings and anticipated rating upgrades. Potentially, one can test whether covenant violations lead to less harsh outcomes during these periods.

More broadly, the market-financing stage can be interpreted as loan sales and securitization.[37] In our model, two kinds of loans may be sold: (1) bad loans that banks try to offload, and (2) good loans for which the borrowers seek cheaper credit in order to relax their borrowing constraints. Existing evidence on loan sales and loan quality is mixed. Dahiya, Puri, and Saunders (2003) find negative announcement effects on loan sales, with almost half of borrowers later filing for bankruptcy. Interestingly, these firms are not the worst-performing firms at the time of loan sales, based on public information such as return on assets, investment, and leverage, suggesting the presence of negative private information in loan sales. By contrast, Drucker and Puri (2009) find that sold loans do not decline in quality. Gande and Saunders (2012) find that a borrowing firm's stock price experiences a positive increase on the first day of its loan being traded in the secondary market, driven by the relaxed financial constraint.[38] As acknowledged by Gande and Saunders (2012), in the sample of Dahiya, Puri, and Saunders (2003), most original lenders terminated their lending relationships after the loan sales. Existing studies document more dubious loans being originated (Keys et al. (2010), Bord and Santos (2015))

[36] They also show that relationship banks strategically downgrade high-quality borrowers' ratings.

[37] Note that we do not model the security-design problem.

[38] Also see Jiang, Nelson, and Vytlacil (2013), who show that loans remaining on a lender's balance sheet ex post have higher delinquency rates than those sold. Their explanation is different though.

under securitization, such as Collateralized Loan Obligations (CLOs) and Collateralized Debt Obligations (CDOs).[39] Our paper predicts that as the financial market develops with the rise of securitization and loan sales (or, equivalently, an improvement in bond market liquidity), zombie lending can be a more secular phenomenon. Potentially, one can verify this pattern using cross-sectional or time-series data.

V. Concluding Remarks

This paper offers a novel explanation for the common phenomenon of zombie lending. In particular, we introduce private learning into a banking model and argue that in a dynamic lending relationship, zombie lending is inevitable but self-limiting. We show that the length of the zombie-lending period is affected by various factors such as the cost of bank and market financing, as well as the entrepreneur's financial constraint. Moreover, we show that in the dynamic lending relationship, the bank has incentives to undersupply effort in producing information.

Our key insights are robust to alternative assumptions. In practice, information about the borrower's quality probably arrives in multiple rounds and is imperfect during each round. The key insights will go through under this alternative assumption. In our model, the entrepreneur and the bank have incentives to conduct zombie lending if they know with certainty that the project is bad. If, instead, they know that the project is likely to be bad (but not with certainty), the cost of rolling over bad loans would be lower, so the incentives to conduct zombie lending should be even stronger. Moreover, even though we do not directly model collateral, L, the liquidation value of the project, can be interpreted as the collateral value that is redeployed for alternative uses (Benmelech (2009)). Kermani and Ma (2020) estimate the liquidation recovery rates of assets among U.S. nonfinancial firms across industries. In particular, one can think of $R - L$ as the additional cash flows generated if the project succeeds; if the project fails, we assume that the collateral value is wiped out. In this sense, the intuition of zombie lending carries over once we introduce the role of collateral. In fact, our results continue to go through under weaker assumptions, for instance, if we assume that the project generates no cash flows if it fails, but the collateral value also falls to $\xi L > 0$, where $\xi \in (0, 1)$. For ξ sufficiently small, all of our results should carry over. Finally, our extension with exogenous lending relationship breakups can be broadly interpreted as shocks to bank capital (Parlour and Plantin (2008)). Results in Section IV.A show that the key insights will carry over if bank equity is introduced.

Zombie lending emerges in our paper due to the substitution between the relationship bank and market-based lenders. An interesting extension would be to introduce com-

[39] Benmelech, Dlugosz, and Ivashina (2012) and Begley and Purnanandam (2017) find the opposite results during a different time period.

plementarity between banks and the market as in Song and Thakor (2010). Moreover, we do not explicitly model interbank competition (Boot and Thakor (2000)). Interbank competition does not change any result in the context of our model, because the new bank is as uninformed as market-based lenders. Studying the tradeoffs of developing multiple lending relationships would be interesting. As Farinha and Santos (2002) document, a young firm could initiate multiple relationships, because the incumbent bank is unwilling to extend credit after poor performance.

Appendix: Proofs

Let us first supplement the definition for the good and bad types' value functions:

$$V_t^g = \max_{\substack{\tau^g \geq t, \\ s.t.\ B^g_{\tau^g} \geq L}} \mathbb{E}_{t^-}\left\{ \int_t^{\tau^g} e^{-r(s-t)} c ds + e^{-r(\tau^g - t)} \left[\mathbb{1}_{\tau^g \geq \tau_\phi} R + \mathbb{1}_{\tau^g < \tau_\phi} \max\{L, \bar{V}^g_{\tau^g}\} \right] \right\}$$

$$V_t^b = \max_{\substack{\tau^b \geq t, \\ s.t.\ B^b_{\tau^b} \geq L}} \mathbb{E}_{t^-}\left\{ \int_t^{\tau^b} e^{-r(s-t)} c ds + e^{-r\left(\tau^b - t\right)} \left[\mathbb{1}_{\tau^b \geq \tau_\phi} \theta R + \mathbb{1}_{\tau^b \geq \tau_\eta} \cdot 0 \right.\right.$$
$$\left.\left. + \mathbb{1}_{\tau^b \leq \min\left\{\tau_\phi, \tau_\eta\right\}} \max\left\{L, \bar{V}^b_{\tau^b}\right\} \right] \right\}$$

$$B_t^g = \mathbb{E}_{t^-}\left\{ \int_t^{\tau^g} e^{-r(s-t)} c ds + e^{-r(\tau^g - t)} \left[\mathbb{1}_{\tau^g \geq \tau_\phi} F + \mathbb{1}_{\tau^g < \tau_\phi} \max\left\{L, \min\left\{\bar{V}^g_{\tau^g}, F\right\}\right\} \right] \right\}$$

$$B_t^b = \mathbb{E}_{t^-}\left\{ \int_t^{\tau^b} e^{-r(s-t)} c ds + e^{-r\left(\tau^b - t\right)} \left[\mathbb{1}_{\tau^b \geq \tau_\phi} \theta F + \mathbb{1}_{\tau^b \geq \tau_\eta} 0 \right.\right.$$
$$\left.\left. + \mathbb{1}_{\tau^b < \min\left\{\tau_\phi, \tau_\eta\right\}} \max\left\{L, \min\left\{\bar{V}^b_{\tau^b}, F\right\}\right\} \right] \right\}.$$

A. *Proof of Proposition 2*

Let us first prove that the No Deals condition and the belief monotonicity requirement imply smooth pasting at $t = t_g$. That is,

$$\dot{V}^g_{t_g} = \dot{D}_t = \dot{q}_t (D^g - D^b).$$

The proof follows closely the proof of Theorem 5.1 in Daley and Green (2012). No Deals and the value matching condition immediately imply that $\dot{V}^g_{t_g} \leq \dot{D}_t = \dot{q}_t \left(D^g - D^b\right)$. Suppose that $\dot{V}^g_{t_g} < \dot{q}_t \left(D^g - D^b\right)$ instead. In this case, consider a deviation in which the good types wait until $t_g + \epsilon$ to refinance with the market, where ϵ is sufficiently small. Belief monotonicity implies that $q_{t_g+\epsilon}$ is at least (approximately) $q_{t_g} + \eta q_{t_g}\left(1 - q_{t_g}\right)\epsilon$, which shows that the good types have strict incentives to wait until $t_g + \epsilon$.

By applying the smooth-pasting and value-matching conditions for type g at the market financing time $t = t_g$, we get

$$\dot{\bar{V}}^g_t = \left(D^g - D^b\right) \eta q_t \left(1 - q_t\right).$$

Using the HJB for type g during $[t_b, t_g]$ and letting $\bar{q} = q_{t_g}$,

$$\begin{aligned}(r+\phi)\,V_t^g &= \dot{V}_t^g + c + \phi R \\ \Rightarrow \phi\,(R-F) + (r+\phi)\left[\bar{q}D^g + (1-\bar{q})\,D^b\right] &= \left(D^g - D^b\right)\eta\bar{q}\,(1-\bar{q}) + c + \phi R.\end{aligned}$$

Next, we show that given Assumption 4, there is only one root on $[0,1]$, which corresponds to the maximal root of the quadratic equation. First, we evaluate the difference between the left-hand side (LHS) and the right-hand side (RHS), $LHS - RHS$, at $\bar{q} = 0$:

$$(r+\phi)\left(D^b - \frac{\delta+\phi}{r+\phi}D^g\right) < 0.$$

Next, we evaluate $LHS - RHS$ at $\bar{q} = 1$:

$$(r+\phi)\left(D^g - \frac{c+\phi F}{r+\phi}\right) = (r+\phi)\left(\frac{c+\phi F}{\delta+\phi} - \frac{c+\phi F}{r+\phi}\right) > 0.$$

So we can conclude that there is only one root on $[0,1]$. Next, we rewrite the quadratic equation for $\bar{q}$ as

$$\bar{q}^2 - \left(1 - \frac{r+\phi}{\eta}\right)\bar{q} + \frac{r+\phi}{\eta}\left(\frac{D^b}{D^g - D^b} - \frac{\delta+\phi}{r+\phi}\frac{D^b}{D^g - D^b}\right) = 0.$$

Note that the minimum of the quadratic function is attained at

$$q^{\min} \equiv \frac{1}{2}\left(1 - \frac{r+\phi}{\eta}\right), \tag{A1}$$

and that $\bar{q} > q^{\min}$. Below we use the observation that $\bar{q} > q^{\min}$ to verify the optimality decisions by different types in equilibrium.

The next step is to solve for the length of $t_g - t_b$. Let

$$\bar{D} = \bar{q}D^g + (1-\bar{q})\,D^b$$

and

$$\bar{V}_{t_g}^b = \bar{D} + \frac{\phi\theta\,(R-F)}{r+\phi+\eta}.$$

Using the boundary condition for the bad type at time t_b, $V_{t_b}^b = L$, together with the type-b's HJB equation on $[t_b, t_g]$,

$$(r+\phi+\eta)\,V_t^b = \dot{V}_t^b + c + \phi\theta R,$$

we obtain

$$t_g - t_b = \frac{1}{r+\phi+\eta}\log\left(\frac{\bar{V}^b_{t_g} - PV^b_r}{L - PV^b_r}\right).$$

From here, we can find t_b using the equation

$$\bar{q} = \frac{q_0}{q_0 + (1-q_0)e^{-(\lambda+\eta)t_b}e^{-\eta(t_g-t_b)}},$$

which yields

$$t_b = \frac{1}{\lambda+\eta}\left[\log\left(\frac{1-q_0}{q_0}\frac{\bar{q}}{1-\bar{q}}\right) - \eta(t_g - t_b)\right].$$

Optimality of the Good Type's Strategy. We need to verify that it is indeed optimal for type g to obtain market financing at time t_g. The HJB equation for the high type on $[0, t_g)$ is

$$(r+\phi)V^g_t = \dot{V}^g_t + c + \phi R.$$

To verify that it is not optimal to delay market financing, we need to verify that the following inequality holds for any $t > t_g$:

$$(r+\phi)\bar{V}^g_t \geq \dot{\bar{V}}^g_t + c + \phi R.$$

To verify that it is not optimal for the good type to seek market financing before time t_g, we need to verify that for any $t < t_g$,

$$V^g_t \geq \bar{V}^g_t.$$

We proceed to verify each of these inequalities. First, we verify the optimality for $t \geq t_g$. Define

$$\begin{aligned}
G_t &= (r+\phi)\bar{V}^g_t - \dot{\bar{V}}^g_t - c - \phi R \\
&= (r+\phi)\left(D_t + \frac{\phi\,(R-F)}{r+\phi}\right) - \dot{D}_t - c - \phi R \\
&= (r+\phi)D_t - \dot{D}_t - c - \phi F.
\end{aligned}$$

By construction, $G_{t_g} = 0$, so it is enough to show that $\dot{G}_t \geq 0$ for $t > t_g$. This amounts to verifying that

$$\dot{G}_t = (r+\phi)\dot{D}_t - \ddot{D}_t \geq 0.$$

Substituting the expressions for D_t, we get

$$\dot{G}_t = (D^g - D^b)\left[(r+\phi)\dot{q}_t - \ddot{q}_t\right].$$

In the last region in which $t \geq t_g$, we have $\dot{q}_t = \eta q_t(1-q_t)$, so we get that

$$\dot{G}_t = (D^g - D^b)\left[r + \phi - \eta(1-2q_t)\right]\dot{q}_t.$$

The conclusion that $\dot{G}_t > 0$ follows from the inequality $\bar{q} > q^{\min}$, where $q^{\min}$ is defined in equation (A1). Next, we verify the optimality for $t < t_g$. Define $H_t \equiv V_t^g - \bar{V}_t^g$. The first step is to show that H_t single-crosses zero from above. We have that

$$\begin{aligned}
\dot{H}_t &= \dot{V}_t^g - \dot{\bar{V}}_t^g \\
&= (r+\phi)V_t^g - c - \phi R - (D^g - D^b)\dot{q}_t \\
&= (r+\phi)H_t + (r+\phi)\bar{V}_t^g - c - \phi R - (D^g - D^b)\dot{q}_t.
\end{aligned}$$

Hence, a sufficient condition is

$$\dot{H}_t\Big|_{H_t=0} = (r+\phi)\bar{V}_t^g - c - \phi R - (D^g - D^b)\dot{q}_t < 0$$

on $(0, t_g)$, which requires that

$$(r+\phi)[q_t D^g + (1-q_t)D^b] < (D^g - D^b)\dot{q}_t + c + \phi F.$$

Since we have $\dot{q}_t \geq \eta q_t\,(1-q_t)$ and $\bar{q} > q^{\min}$, it follows that

$$(r+\phi)[qD^g + (1-q)D^b] < (D^g - D^b)\eta q\,(1-q) + c + \phi F,$$

for all $0 < q < \bar{q}$, which means that $\dot{H}_t\Big|_{H_t=0} < 0$ for $t < t_g$. From here we can conclude that $V_t^g \geq \bar{V}_t^g$ for $t < t_g$.

Optimality of the Bad Type's Strategy. The strategy of the low type is optimal if for any $t < t_b$, $(r+\phi+\eta)\,L \geq c + \phi\theta R \Rightarrow L \geq PV_r^b$ and for any $t \geq t_b$, $V_t^b \geq L$. To verify that $V_t^b \geq L$ for $t > t_b$, notice that on (t_b, t_g), the value function satisfies

$$(r+\phi+\eta)\,V_t^b = \dot{V}_t^b + c + \phi\theta R.$$

This equation can be written as

$$(r+\phi+\eta)\,(V_t^b - L) = \dot{V}_t^b + c + \phi\theta R - (r+\phi+\eta)\,L.$$

Letting $G_t = V_t^b - L$, we obtain the equation

$$\dot{G}_t = (r+\phi+\eta)\left(G_t + L - PV_r^b\right),\ G_{t_b} = 0.$$

Clearly, $\dot{G}_t\big|_{G_t=0} > 0$ so that $G_t = V_t^b - L \geq 0$ for all $t \geq t_b$.

Optimality of the Uninformed Type's Strategy. Next, we verify that the uninformed type is better off rolling over at time $t < t_b$ rather than liquidating. First, we solve for the continuation value of the uninformed type at any time $t < t^b$. For $t \in (0, t_b)$, we have that

$$\begin{aligned}(r + \phi + \lambda + (1 - \mu_t)\eta)\, V_t^u =& \dot{V}_t^u + c + \phi\left[\mu_t + (1 - \mu_t)\,\theta\right] R \\ &+ \lambda\left[\mu_t V_t^g + (1 - \mu_t)\, L\right] \\ (r + \phi)\, V_t^g =& \dot{V}_t^g + c + \phi R.\end{aligned}$$

Solving backwards starting at time t_b, we get

$$\begin{aligned}V_t^u = \int_t^{t_b} & e^{-(r+\phi+\lambda)(s-t) - \int_t^s \eta(1-\mu_u)du} \left(c + \phi\left[\mu_s + (1 - \mu_s)\,\theta\right] R + \lambda\left[\mu_s V_s^g + (1 - \mu_s)\, L\right]\right) ds \\ &+ e^{-(r+\phi+\lambda)(t_b-t) - \int_t^{t_b} \eta(1-\mu_u)du} V_{t_b}^u.\end{aligned}$$

Substituting the relation

$$\int_t^s \eta(1 - \mu_s)ds = \int_t^s \frac{\dot{\mu}_s}{\mu_s} ds = \log(\mu_s/\mu_t)$$

and the continuation value of the good type

$$V_t^g = \frac{c + \phi R}{r + \phi}\left(1 - e^{-(r+\phi)(t_b-t)}\right) + e^{-(r+\phi)(t_b-t)} V_{t_b}^g,$$

we obtain

$$\begin{aligned}V_t^u = & \mu_t\left[PV_r^g + e^{-(r+\phi)(t_b-t)}\left(V_{t_b}^g - PV_r^g\right)\right] \\ &+ (1 - \mu_t)\left[\frac{c + \phi\theta R + \lambda L}{r + \phi + \lambda + \eta} + e^{-(r+\phi+\lambda+\eta)(t_b-t)}\left(L - \frac{c + \phi\theta R + \lambda L}{r + \phi + \lambda + \eta}\right)\right]. \qquad \text{(A2)}\end{aligned}$$

It is convenient to express the continuation value of the uninformed type in terms of the uninformed's belief μ_t. Let $t(\mu) = -\frac{1}{\eta}\log\left(\frac{q_0}{1-q_0}\frac{1-\mu}{\mu}\right)$ be the time at which the belief is μ. μ_b is given by $t(\mu_b) = t_b$ so

$$t(\mu_b) - t(\mu) = -\frac{1}{\eta}\log\left(\frac{1 - \mu_b}{\mu_b}\frac{\mu}{1 - \mu}\right).$$

Substituting $t(\mu_b) - t(\mu)$, we get

$$V^u(\mu) = \mu \left[PV_r^g + \left(\frac{1-\mu_b}{\mu_b} \frac{\mu}{1-\mu} \right)^{\frac{r+\phi}{\eta}} \left(V_{t_b}^g - PV_r^g \right) \right]$$
$$+ (1-\mu) \left[\frac{c + \phi\theta R + \lambda L}{r + \phi + \lambda + \eta} + \left(\frac{1-\mu_b}{\mu_b} \frac{\mu}{1-\mu} \right)^{1+\frac{r+\phi+\lambda}{\eta}} \left(L - \frac{c + \phi\theta R + \lambda L}{r + \phi + \lambda + \eta} \right) \right]. \quad \text{(A3)}$$

Letting $z \equiv \mu/(1-\mu)$, we have that $V^u(\mu) \geq L$ if

$$z \left[PV_r^g - L + \left(\frac{z}{z_b} \right)^{\frac{r+\phi}{\eta}} \left(V_{t_b}^g - PV_r^g \right) \right] - \frac{r + \phi + \eta}{r + \phi + \lambda + \eta} \left[1 - \left(\frac{z}{z_b} \right)^{1+\frac{r+\phi+\lambda}{\eta}} \right] \left(L - \frac{c + \phi\theta R}{r + \phi + \lambda} \right) > 0.$$

The LHS is increasing in z (so $V^u(\mu)$ is increasing in μ), and hence $V^u(\mu) \geq L$ for all $\mu \in [q_0, \mu_b]$ only if $V^u(q_0) \geq L$.

No Deals for the Uninformed. Finally, we need to verify that the no deals condition holds for the uninformed type. This is immediate when $\eta = 0$, but requires verification when $\eta > 0$. No deals requires that

$$V_t^u \geq \tilde{D}_t + \mu_t \frac{\phi(R-F)}{r+\phi} + (1-\mu_t) \frac{\phi\theta(R-F)}{r+\phi+\eta},$$

where $\tilde{D}_t$ is the value of debt if the uninformed type is pooled with the bad type. In particular,

$$\tilde{D}_t = \tilde{q}_t D^g + (1-\tilde{q}_t) D^b,$$

where $\tilde{q}_t$ is the belief conditional on being either uninformed or bad. For $t < t_b$, the probability of being bad is zero, so the probability of the project being good conditional on being either bad or uninformed is given by

$$\tilde{q}_t = \mu_t = \frac{q_0}{q_0 + (1-q_0)e^{-\eta t}}.$$

For $t \in (t_b, t_g)$ we have

$$\tilde{q}_t = \mu_t \frac{\pi_t^u}{1 - \pi_t^g},$$

where

$$\pi_t^u = \frac{\left(q_0 + (1-q_0)e^{-\eta t}\right) e^{-\lambda t}}{q_0 + (1-q_0)e^{-(\lambda+\eta)t_b} e^{-\eta(t-t_b)}}$$
$$\pi_t^g = \frac{q_0(1-e^{-\lambda t})}{q_0 + (1-q_0)e^{-(\lambda+\eta)t_b} e^{-\eta(t-t_b)}},$$

so

$$\frac{\pi_t^u}{1-\pi_t^g} = \frac{\left(q_0 + (1-q_0)e^{-\eta t}\right) e^{-\lambda t}}{q_0 e^{-\lambda t} + (1-q_0)e^{-(\lambda+\eta)t_b} e^{-\eta(t-t_b)}}$$

and

$$\tilde{q}_t = \frac{q_0 e^{-\lambda t}}{q_0 e^{-\lambda t} + (1-q_0)e^{-(\lambda+\eta)t_b} e^{-\eta(t-t_b)}} = \frac{q_0}{q_0 + (1-q_0)e^{-\lambda t_b} e^{(\lambda-\eta)t}}.$$

From here we get that $\tilde{D}_t$ is decreasing in time only if $\lambda > \eta$. For any $t \in [t_b, t_g]$, the continuation value of the uninformed type is given by

$$V_t^u = \mu_t V_t^g + (1-\mu_t) V_t^b.$$

So the no-deals condition on (t_b, t_g) can be written as

$$\mu_t \left(V_t^g - \frac{\phi(R-F)}{r+\phi} \right) + (1-\mu_t) \left(V_t^b - \frac{\phi\theta(R-F)}{r+\phi+\eta} \right) \geq \tilde{D}_t,$$

where the LHS is increasing in t.

CLAIM 1:

$$\mu_t \left(V_t^g - \frac{\phi(R-F)}{r+\phi} \right) + (1-\mu_t) \left(V_t^b - \frac{\phi\theta(R-F)}{r+\phi+\eta} \right)$$

is increasing in time.

Proof: To show that the expression in the proposition is increasing in time, it is sufficient to show that

$$V_t^g - V_t^b \geq \frac{\phi(R-F)}{r+\phi} - \frac{\phi\theta(R-F)}{r+\phi+\eta}.$$

At time t_g, we have

$$V_t^g - V_t^b = \frac{\phi(R-F)}{r+\phi} - \frac{\phi\theta(R-F)}{r+\phi+\eta},$$

while at any time $t < t^g$, we have

$$\dot{V}_t^g - \dot{V}_t^b = (r+\phi)(V_t^g - V_t^b) - \eta V_t^b - \phi(1-\theta)R.$$

Solving backward in time starting at t_g, we get

$$V_t^g - V_t^b = \frac{\eta}{r+\phi} \frac{c+\phi\theta R}{r+\phi+\eta} \left(1 - e^{-(r+\phi)(t_g-t)}\right) + e^{-(r+\phi)(t_g-t)} \left(1 - e^{-\eta(t_g-t)}\right) \left(\bar{V}_{t_g}^b - \frac{c+\phi\theta R}{r+\phi+\eta} \right)$$
$$+ \frac{\phi(1-\theta)R}{r+\phi} \left(1 - e^{-(r+\phi)(t_g-t)}\right) + e^{-(r+\phi)(t_g-t)} \left(\frac{\phi(R-F)}{r+\phi} - \frac{\phi\theta(R-F)}{r+\phi+\eta} \right).$$

Hence, we get that

$$V_t^g - V_t^b - \left(\frac{\phi(R-F)}{r+\phi} - \frac{\phi\theta(R-F)}{r+\phi+\eta}\right) = e^{-(r+\phi)(t_g-t)}\left(1-e^{-\eta(t_g-t)}\right)\left(\bar{V}_{t_g}^b - \frac{c+\phi\theta R}{r+\phi+\eta}\right)$$
$$+\frac{\left(1-e^{-(r+\phi)(t_g-t)}\right)}{r+\phi+\eta}\left(\frac{\eta}{r+\phi}c + \phi(1-\theta)F + \frac{\eta}{r+\phi}\phi F\right) > 0.$$

It follows immediately from the fact that μ_t, V_t^b, and V_t^g are increasing in time that

$$\mu_t\left(V_t^g - \frac{\phi(R-F)}{r+\phi}\right) + (1-\mu_t)\left(V_t^b - \frac{\phi\theta(R-F)}{r+\phi+\eta}\right)$$

is also increasing in time. □

If $\lambda > \eta$, then the previous claim implies that it is enough to verify the uninformed type's no-deals condition at time t_b to guarantee that it is satisfied for all $t \in [t_b, t_g]$. In this case, we only need to verify that

$$\mu_{t_b}\left(V_{t_b}^g - \frac{\phi(R-F)}{r+\phi}\right) + (1-\mu_{t_b})\left(L - \frac{\phi\theta(R-F)}{r+\phi+\eta}\right) \geq \tilde{D}_{t_b}. \tag{A4}$$

At time t_b, we have that

$$\tilde{q}_{t_b} = \frac{q_0}{q_0 + (1-q_0)e^{-\eta t_b}} = \mu_{t_b},$$

and thus we can write condition (A4) as

$$\mu_{t_b}\left(V_{t_b}^g - D^g - \frac{\phi(R-F)}{r+\phi}\right) + (1-\mu_{t_b})\left(L - D^b - \frac{\phi\theta(R-F)}{r+\phi+\eta}\right) \geq 0. \tag{A5}$$

Therefore, we are only left to verify No Deals on $t \in [0, t_b]$. Because $\tilde{q}_t = \mu_t$ on $(0, t_b)$, the no-deals condition for the uninformed type on $(0, t_b)$ amounts to verifying that

$$V_t^u \geq \mu_t\left(D^g + \frac{\phi(R-F)}{r+\phi}\right) + (1-\mu_t)\left(D^b + \frac{\phi\theta(R-F)}{r+\phi+\eta}\right).$$

Using equation (A3), we can write the uninformed's no-deal condition as

$$F(z) \equiv z\left(\frac{z}{z_b}\right)^{\frac{r+\phi}{\eta}}\left(V_{t_b}^g - PV_r^g\right) + \left(\frac{z}{z_b}\right)^{1+\frac{r+\phi+\lambda}{\eta}}\left(L - \frac{c+\phi\theta R+\lambda L}{r+\phi+\lambda+\eta}\right)$$
$$-z\left(D^g + \frac{\phi(R-F)}{r+\phi} - PV_r^g\right) + \frac{c+\phi\theta R+\lambda L}{r+\phi+\lambda+\eta} - \frac{\phi\theta(R-F)}{r+\phi+\eta} - D^b \geq 0.$$

It can be easily verified that $F(z)$ is convex and that its first derivative is given by

$$F'(z) = \left(1 + \frac{r+\phi}{\eta}\right)\left(\frac{z}{z_b}\right)^{\frac{r+\phi}{\eta}} \left[\left(V^g_{t_b} - PV^g_r\right) + \frac{1}{z_b}\left(L - PV^b_r\right)\right] - \left(D^g + \frac{\phi(R-F)}{r+\phi} - PV^g_r\right).$$

To verify the no-deals condition, we need to consider the case in which the minimum of $F(z)$ is on the boundary of $[z_0, z_b]$ as well as the case in which it is in the interior. Because $F(z)$ is convex, the previous three cases correspond to: 1) if $F'(z_0) \geq 0$, then F is increasing on $[z_0, z_b]$ so it is sufficient to check that $F(z_0) \geq 0$; 2) if $F'(z_b) \leq 0$, then $F(z)$ is decreasing on $[z_0, z_b]$ so it is sufficient to check that $F(z_b) \geq 0$; and 3) if $F'(z_0) < 0 < F'(z_b)$, then F attains its minimum at $z^{\min}$ in the interior of $[z_0, z_b]$, and we need to verify that $F(z^{\min}) \geq 0$. Notice that $F'(z) > 0$ when $\eta \to 0$, so for η sufficiently small, the uninformed no deals condition reduces to $F(z_0) \geq 0$.

Case 1: $F'(z_0) \geq 0$. This is the case when

$$\left(\frac{z_0}{z_b}\right)^{\frac{r+\phi}{\eta}} \geq \frac{\eta}{r+\phi+\eta} \frac{\left(D^g + \frac{\phi(R-F)}{r+\phi} - PV^g_r\right)}{\left(V^g_{t_b} - PV^g_r\right) + \frac{1}{z_b}\left(L - PV^b_r\right)}.$$

In contrast, if $F'(z_0) \geq 0$, then the uninformed type's no-deals condition is satisfied if $F(z_0) \geq 0$. We have that $F'(z_0) \geq 0$ if

$$\left(\frac{z_0}{z_b}\right)^{\frac{r+\phi}{\eta}} > \frac{\eta}{r+\phi+\eta} \frac{\left(D^g + \frac{\phi(R-F)}{r+\phi} - PV^g_r\right)}{\left(V^g_{t_b} - PV^g_r\right) + \frac{1}{z_b}\left(L - PV^b_r\right)}.$$

In this case, the uninformed's no-deals condition is

$$\left(\frac{z_0}{z_b}\right)^{\frac{r+\phi}{\eta}} \left(V^g_{t_b} - PV^g_r\right) + \left[\left(\frac{z_0}{z_b}\right)^{1+\frac{r+\phi+\lambda}{\eta}} - 1\right] \frac{r+\phi+\eta}{r+\phi+\lambda+\eta}\left(L - PV^b_r\right) \geq z_0\left(D^g + \frac{\phi(R-F)}{r+\phi} - PV^g_r\right) + D^b + \frac{\phi\theta(R-F)}{r+\phi+\eta} - L,$$

which holds for η sufficiently small. This condition is captured by $\eta < \bar{\eta}$.

For completeness, we also specify the conditions for the two other cases. Note that for η sufficiently small, these two cases will not show up.

Case 2: $F'(z_b) \leq 0$. This is the case when

$$1 \leq \frac{\eta}{r+\phi+\eta} \frac{\left(D^g + \frac{\phi(R-F)}{r+\phi} - PV_r^g\right)}{\left(V_{t_b}^g - PV_r^g\right) + \frac{1}{z_b}\left(L - PV_r^b\right)}.$$

If the previous inequality is satisfied, the uninformed type's no-deals condition reduces to $F(z_b) \geq 0$, which can be written as

$$V_{t_b}^g - PV_r^g \geq z_b \left(D^g + \frac{\phi(R-F)}{r+\phi} - PV_r^g\right) + D^b + \frac{\phi\theta(R-F)}{r+\phi+\eta} - L.$$

Case 3: $F'(z_0) < 0 < F'(z_b)$. Finally, if

$$\left(\frac{z_0}{z_b}\right)^{\frac{r+\phi}{\eta}} < \frac{\eta}{r+\phi+\eta} \frac{\left(D^g + \frac{\phi(R-F)}{r+\phi} - PV_r^g\right)}{\left(V_{t_b}^g - PV_r^g\right) + \frac{1}{z_b}\left(L - PV_r^b\right)} < 1,$$

then $F(z)$ attains its minimum in the interior of (z_0, z_b) and we need to check the no-deals condition at its minimum. Solving for the first-order condition, we find that $z^{\min} = \arg\min_z F(z)$ is

$$\left(\frac{z^{\min}}{z_b}\right) = \left[\frac{\left(D^g + \frac{\phi(R-F)}{r+\phi} - PV_r^g\right)}{\left(1 + \frac{r+\phi}{\eta}\right)\left[\left(V_{t_b}^g - PV_r^g\right) + \frac{1}{z_b}\left(L - PV_r^b\right)\right]}\right]^{\frac{\eta}{r+\phi}}.$$

Substituting $z^{\min}$ in $F(z)$, we find that the no-deals condition for the uninformed type in this case is

$$\frac{(r+\phi)\left(V_{t_b}^g - PV_r^g\right) - \frac{(r+\phi+\eta)(r+\phi+\lambda)}{r+\phi+\lambda+\eta}\frac{1}{z_b}\left(L - PV_r^b\right)}{\left(1 + \frac{r+\phi}{\eta}\right)^{1+\frac{\eta}{r+\phi}}\left[\left(V_{t_b}^g - PV_r^g\right) + \frac{1}{z_b}\left(L - PV_r^b\right)\right]^{1+\frac{\eta}{r+\phi}}} \geq \frac{\eta}{z_b} \frac{D^b + \frac{\phi\theta(R-F)}{r+\phi+\eta} - \frac{c+\phi\theta R+\lambda L}{r+\phi+\lambda+\eta}}{\left(D^g + \frac{\phi(R-F)}{r+\phi} - PV_r^g\right)^{1+\frac{\eta}{r+\phi}}}.$$

B. *Proof of Proposition 3*

The financial constraint is only relevant for the rollover decision. The bank will be willing to rollover only if $B_t^b \geq L$. In the equilibrium without the financial constraint, $B_{t_b}^b < V_{t_b}^b = L$. Hence, in the presence of the financial constraint, the boundary condition for t_b is replaced by $B_{t_b}^b$. By direct computation, we get that the bank's continuation value at time t_b is given by

$$B_{t_b}^b = \frac{c+\phi\theta F}{r+\phi+\eta}\left(1 - e^{-(r+\phi+\eta)(t_g - t_b)}\right) + e^{-(r+\phi+\eta)(t_g-t_b)} F.$$

Solving the boundary condition $B^b_{t_b} = L$, we get

$$t_g - t_b = \frac{1}{r+\phi+\eta} \log\left(\frac{F - \frac{c+\phi\theta F}{r+\phi+\eta}}{L - \frac{c+\phi\theta F}{r+\phi+\eta}} \right).$$

The no-deals conditions for the good and uninformed types are the same as in the unconstrained case. Hence, the only step left is to analyze the optimality of the rollover strategy. First, we look at the problem of the low type. In this case, we need to verify that $B^b_t \geq L$ for $t > t_b$, and that it is not optimal to delay liquidation before time t_b. To verify that $B^b_t \geq L$ on (t_b, t_g), notice that

$$(r+\phi+\eta)\, B^b_t = \dot{B}^b_t + c + \phi\theta F,$$

so it follows that

$$\dot{B}^b_t\Big|_{B^b_t = L} = (r+\phi+\eta)\left(L - \frac{c+\phi\theta F}{r+\phi+\eta}\right) > (r+\phi+\eta)\left(L - PV^b_r\right) > 0,$$

which immediately implies that $B^b_t \geq L$ for $t > t_b$. To verify that it is not optimal to delay liquidation on $(0, t_b)$, notice that

$$\dot{B}^b_t + c + \phi\theta F - (r+\phi+\eta)\, B^b_t = c + \phi\theta F - (r+\phi+\eta)\, L < 0,$$

which implies that it is optimal to liquidate for $t < t_b$.

Next, we need to verify that the uninformed type is willing to roll over the loan at time $t \in (0, t_b)$. The continuation value of the uninformed type satisfies the equation

$$\begin{aligned}(r+\phi+\lambda+(1-\mu_t)\eta)\, B^u_t =& \dot{B}^u_t + c + \phi\left[\mu_t + (1-\mu_t)\,\theta\right] F \\ &+ \lambda\left[\mu_t B^g_t + (1-\mu_t)\, L\right] \\ (r+\phi) B^g_t =& \dot{B}^g_t + c + \phi F.\end{aligned}$$

Solving backward in time starting at t_b, we get that for any $t \in [0, t_b]$, the uninformed's continuation value is

$$\begin{aligned}B^u_t = \mu_t &\left[\frac{c+\phi F}{r+\phi} + e^{-(r+\phi)(t_b - t)}\left(B^g_{t_b} - \frac{c+\phi F}{r+\phi}\right)\right] \\ &+ (1-\mu_t)\left[\frac{c+\phi\theta F + \lambda L}{r+\phi+\lambda+\eta} + e^{-(r+\phi+\lambda+\eta)(t_b - t)}\left(L - \frac{c+\phi\theta F + \lambda L}{r+\phi+\lambda+\eta}\right)\right]. \quad \text{(A6)}\end{aligned}$$

Let us rewrite B_t^u in the belief domain:

$$B^u(\mu) = \mu\left[\frac{c+\phi F}{r+\phi} + \left(\frac{1-\mu_b}{\mu_b}\frac{\mu}{1-\mu}\right)^{\frac{r+\phi}{\eta}}\left(B_{t_b}^g - \frac{c+\phi F}{r+\phi}\right)\right]$$
$$+ (1-\mu)\left[\frac{c+\phi\theta F+\lambda L}{r+\phi+\lambda+\eta} + \left(\frac{1-\mu_b}{\mu_b}\frac{\mu}{1-\mu}\right)^{1+\frac{r+\phi+\lambda}{\eta}}\left(L - \frac{c+\phi\theta F+\lambda L}{r+\phi+\lambda+\eta}\right)\right]. \quad \text{(A7)}$$

The condition $B_t^u \geq L$ can be written in terms of the likelihood ratio $z \equiv \mu/(1-\mu)$ as

$$z\left[\frac{c+\phi F}{r+\phi} - L + \left(\frac{z}{z_b}\right)^{\frac{r+\phi}{\eta}}\left(B_{t_b}^g - \frac{c+\phi F}{r+\phi}\right)\right]$$
$$- \frac{r+\phi+\eta}{r+\phi+\lambda+\eta}\left[1 - \left(\frac{z}{z_b}\right)^{1+\frac{r+\phi+\lambda}{\eta}}\right]\left(L - \frac{c+\phi\theta F}{r+\phi+\eta}\right) \geq 0.$$

The LHS is increasing in z, so it is enough to verify that $B_0^u \geq L$.

C. *Proof of Proposition 4 and Corollary 3*

Proof: The result on $\bar{q}$ naturally follows by plugging $\eta = 0$ into Propositions 2 and 3. The results on $\bar{q}$ and $t_g - t_b$ follow from Assumptions 1 and 4. □

D. *Proof of Proposition 5*

Proof: Given the boundary conditions, we can show that

$$B_{t_b}^g = \frac{c+\phi F}{r+\phi} + e^{-(r+\phi)(t_g-t_b)}\left(F - \frac{c+\phi F}{r+\phi}\right),$$

whereas $B_{t_b}^b = L$. When $t \in [0, t_b]$, the HJB satisfies

$$(r+\phi+\lambda)\, B_t^u = \dot{B}_t^u + c + \phi\left[q_0 + (1-q_0)\,\theta\right] F + \lambda\left[q_0 B_t^g + (1-q_o)\, L\right].$$

Solving this ordinary differential equation (ODE), we can write B_0^u in terms of primitives,

$$B_0^u = q_0\left[\frac{c+\phi F}{r+\phi} + \left(\frac{1-\bar{q}}{\bar{q}}\frac{q_0}{1-q_0}\right)^{\frac{r+\phi}{\lambda}}\left(\frac{L - \frac{c+\phi\theta F}{r+\phi}}{F - \frac{c+\phi\theta F}{r+\phi}}\right)\frac{rF-c}{r+\phi}\right]$$
$$+ (1-q_0)\left[\frac{c+\phi\theta F+\lambda L}{r+\phi+\lambda} + \left(\frac{1-\bar{q}}{\bar{q}}\frac{q_0}{1-q_0}\right)^{1+\frac{r+\phi}{\lambda}}\left(L - \frac{c+\phi\theta F+\lambda L}{r+\phi+\lambda}\right)\right].$$

Finally,

$$
\begin{aligned}
\frac{dB_0^u}{d\frac{1-\bar{q}}{\bar{q}}} = & \; q_0^{1+\frac{r+\phi}{\lambda}} \left(1-q_0\right)^{-\frac{r+\phi}{\lambda}} \times \left(\frac{1-\bar{q}}{\bar{q}}\right)^{\frac{r+\phi}{\lambda}-1} \\
& \times \left(\frac{L-\frac{c+\phi\theta F}{r+\phi}}{F-\frac{c+\phi\theta F}{r+\phi}}\right) \frac{rF-c}{r+\phi}\frac{r+\phi}{\lambda} + \left(L-\frac{c+\phi\theta F+\lambda L}{r+\phi+\lambda}\right)\left(1+\frac{r+\phi}{\lambda}\right)\left(\frac{1-\bar{q}}{\bar{q}}\right),
\end{aligned}
$$

which is positive if $\left(L-\frac{c+\phi\theta F+\lambda L}{r+\phi+\lambda}\right)\left(1+\frac{r+\phi}{\lambda}\right)\left(\frac{1-\bar{q}}{\bar{q}}\right) > \left(\frac{L-\frac{c+\phi\theta F}{r+\phi}}{F-\frac{c+\phi\theta F}{r+\phi}}\right)\frac{c-rF}{r+\phi}\frac{r+\phi}{\lambda}$. Since $\bar{q}$ increases with δ, $\frac{dB_0^u}{d\frac{1-\bar{q}}{\bar{q}}} > 0$ and equivalently $\frac{dB_0^u}{d\delta} < 0$ if δ is sufficiently small, whereas $\frac{dB_0^u}{d\delta} < 0$ if δ gets sufficiently large. □

E. *Proof of Proposition 7 and Corollary 4*

We offer the proof for the case without the financial constraint. The proofs for the case with the financial constraint and Corollary 4 are available in the Internet Appendix.

Define $\underline{t}_a \equiv \frac{1}{\lambda}\log\left(\frac{\bar{q}}{1-\bar{q}}\frac{1-q_0}{q_0}\right)$. The proof is divided into two parts. First, we show that the equilibrium can be characterized by three thresholds, $\{t_a, t_b, t_g\}$. Second, we derive equations determining $\{t_a, t_b, t_g\}$.

Recall that the boundary conditions for t_b and t_g are determined by the informed type b and g, so that endogenous learning will not affect the existence of the three equilibrium regions, as well as the boundary conditions. It only remains to determine the equilibrium learning policy of the uninformed type. The first step is to show that the bank never learns after time t_b. The result on $[t_g, \infty)$ is straightforward, so we prove that there is no learning for $t \in [t_b, t_g)$.

Define

$$
\begin{aligned}
\Psi_t &\equiv \int_t^{t_g} e^{-(r+\phi)(s-t)-\int_t^s \lambda a_u du}\psi a_s ds \\
\Gamma_t &\equiv q_0 B_t^g + (1-q_0)\, B_t^b - B_t^u.
\end{aligned}
$$

Suppose that the bank learns during (t_b, t_g). For any $t \in (t_a, t_g)$, the HJB equation is

$$\left(r+\phi+\frac{1}{m}\right)B_t^u = \dot{B}_t^u + y_tF - \psi a_t + \phi[q_0+(1-q_0)\theta]F + \frac{1}{m}V_t^u + \lambda a_t\Gamma_t \tag{A8a}$$

$$\left(r+\phi+\frac{1}{m}\right)B_t^g = \dot{B}_t^g + y_tF + \phi F + \frac{1}{m}V_t^g \tag{A8b}$$

$$\left(r+\phi+\frac{1}{m}\right)B_t^b = \dot{B}_t^b + y_tF + \phi\theta F + \frac{1}{m}V_t^b. \tag{A8c}$$

The ODE for Γ_t follows

$$\left(r+\phi+\frac{1}{m}+\lambda a_t\right)\Gamma_t=\dot{\Gamma}_t+\psi a_t+\frac{1}{m}(q_0V_t^g+(1-q_0)V_t^b-V_t^u)=\dot{\Gamma}_t+\psi a_t+\frac{1}{m}\Psi_t.$$

Since $\Gamma_{t_g}=0$, it implies that $\lambda\Gamma_t\leq 0<\psi$ for $\forall t\in[t_b,t_g]$. Therefore, the bank never learns on $t\in[t_b,t_g)$.

Next, we prove that it if learning happens at all, then it must happen on $[0,t_a]$ for some $t_a<t_b$. Let $t_a=\sup\{t\leq t_b:\lambda\Gamma_t=\psi\}$. Noticing that $\Gamma_{t_b}=0$, we can conclude that $t_a<t_b$. We want to show that the optimal policy is $a_t=\mathbf{1}_{t<t_a}$. Suppose not. Then there exists t_a' such that $\lambda\Gamma_t<\psi$ on $(t_a'-\epsilon,t_a')$. In particular, consider $t_a'=\sup\{t<t_a:\lambda\Gamma_t<\psi\}$. Consider the region (t_a',t_a). In this region, the bank's HJB equation is

$$\begin{aligned}
\left(r+\phi+\frac{1}{m}\right)B_t^u&=\dot{B}_t^u+y_tF+\phi[q_0+(1-q_0)\theta]F-\psi+\frac{1}{m}V_t^u+\lambda\Gamma_t\\
\left(r+\phi+\frac{1}{m}\right)B_t^g&=\dot{B}_t^g+y_tF+\phi F+\frac{1}{m}V_t^g\\
\left(r+\phi+\frac{1}{m}\right)B_t^b&=\dot{B}_t^b+y_tF+\phi\theta F+\frac{1}{m}L,
\end{aligned}$$

so

$$\left(r+\phi+\frac{1}{m}+\lambda\right)\Gamma_t=\dot{\Gamma}_t+\frac{1}{m}(q_0V_t^g+(1-q_0)L-V_t^u)+\psi. \tag{A9}$$

Let $H_t\equiv(1-q_0)L+q_0V_t^g-V_t^u$. We get

$$\begin{aligned}
\left(r+\phi+\frac{1}{m}+\lambda\right)\Gamma_t&=\dot{\Gamma}_t+\frac{1}{m}H_t+\psi,\ t\in(t_a',t_a)\\
\left(r+\phi+\frac{1}{m}\right)\Gamma_t&=\dot{\Gamma}_t+\frac{1}{m}H_t,\ t\in(t_a,t_b).
\end{aligned}$$

Taking the left and right limits at t_a', we get $\dot{\Gamma}_{t_a-}=\dot{\Gamma}_{t_a+}$, so Γ_t is differentiable at t_a. It follows from the ODE for Γ_t that if $\dot{H}_t\leq 0$ on (t_a',t_b), then Γ_t is a quasi-convex function of t on (t_a',t_b). To show that $\dot{H}_t\leq 0$, we write an ODE for H_t using the HJB equations for V_t^b and V_t^g,

$$(r+\phi+\lambda)\,H_t=\dot{H}_t+(r+\phi)\,(1-q_0)\,L-(1-q_0)(c+\phi\theta R) \tag{A10}$$

$$+\,\psi-\lambda\,(1-q_0)\,(V_t^b-L),\ t\in(t_a',t_a) \tag{A11}$$

$$(r+\phi)\,H_t=\dot{H}_t+(r+\phi)\,(1-q_0)\,L-(1-q_0)(c+\phi\theta R),\ t\in(t_a,t_b), \tag{A12}$$

where $H_{t_b}=(1-q_0)V_{t_b}^b+q_0V_{t_b}^g-V_{t_b}^u=0$. Assumption 1 implies that $\dot{H}_{t_b}<0$. Differentiating equations (A11) and (A12), we get

$$
\begin{aligned}
(r+\phi+\lambda)\,\dot{H}_t &= \ddot{H}_t - \lambda\,(1-q_0)\,\dot{V}_t^b,\ \ t\in(t_a',t_a)\\
(r+\phi)\,\dot{H}_t &= \ddot{H}_t,\ \ t\in(t_a,t_b).
\end{aligned}
$$

It immediately follows that $\dot{H}_t = 0 \Rightarrow \ddot{H}_t \geq 0$ since $\dot{V}_t^b \geq 0$. Hence, $\dot{H}_t$ single-crosses zero from negative to positive, so $\dot{H}_{t_b} < 0 \Rightarrow \dot{H}_t < 0$, $\forall t \in (t_a', t_b)$.

Since Γ_t is quasi-convex on (t_a', t_b), $\Gamma_{t_b} = 0$ and $\Gamma_{t_a} = \psi/\lambda$. It must be the case that $\Gamma_{t_a'} > \psi/\lambda$, which provides the desired contractions. Thus, it must be the case that $\lambda\Gamma_t \geq \psi$ for all $t < t_a$

Having shown that the optimal policy is characterized by $\{t_a, t_b, t_g\}$, we provide a solution and derive parametric assumptions needed to validate it. Note that in the equilibrium characterized by $\{t_a, t_b, t_g\}$, beliefs evolve on $t \in (t_a, t_b)$ according to

$$
\begin{aligned}
\dot{\pi}_t^u &= \frac{1}{m}\pi_t^u\pi_t^b\\
\dot{\pi}_t^g &= \frac{1}{m}\pi_t^g\pi_t^b\\
\dot{\pi}_t^b &= -\frac{1}{m}\pi_t^b\left(1-\pi_t^b\right),
\end{aligned}
$$

which means that the average quality evolves according to

$$
\dot{q}_t = \frac{1}{m}q_t\pi_t^b.
$$

Solving the previous equation starting at time t_a, we obtain that for any $t > t_a$, the average belief is

$$
q_t = q_{t_a}e^{\frac{1}{m}\int_{t_a}^t \pi_s^b ds}. \tag{A13}
$$

The differential equation for π_t^b is decoupled from those for π_t^u and π_t^g, so it can be solved independently to get

$$
\begin{aligned}
\pi_t^b &= \frac{\pi_{t_a}^b}{\pi_{t_a}^b + (1-\pi_{t_a}^b)e^{\frac{1}{m}(t-t_a)}}\\
e^{\frac{1}{m}\int_{t_a}^t \pi_s^b ds} &= \frac{1-\pi_t^b}{1-\pi_{t_a}^b}.
\end{aligned}
$$

Substituting in equation (A13), we get

$$
q_t = \frac{1}{1-\pi_{t_a}^b + \pi_{t_a}^b e^{-\frac{1}{m}(t-t_a)}}q_{t_a}. \tag{A14}
$$

To find $\pi^b_{t_a}$, we use equations (IA9), (IA10), and (IA11) in the Internet Appendix to get

$$\pi^b_{t_a} = 1 - \frac{q_{t_a}}{q_0}\left(q_0 + (1-q_0)e^{-\lambda t_a}\right).$$

Taking the limit of q_t in equation (A14) as $t \to \infty$, we get that $q_t \to \frac{q_{t_a}}{1-\pi^b_{t_a}}$, so $\lim_{t\to\infty} q_t > \bar{q}$ only if

$$t_a \geq \underline{t}_a \equiv \frac{1}{\lambda}\log\left(\frac{\bar{q}}{1-\bar{q}}\frac{1-q_0}{q_0}\right). \tag{A15}$$

Having established a lower bound for t_a, we derive a system of equations for t_a, t_b, t_g. On (t_a, t_b), the bank's continuation value satisfies equation (A8) evaluated at $a_t = 0$. We can solve for Γ_t using the terminal condition $\Gamma_{t_b} = 0$ to get

$$\Gamma_t = \int_t^{t_b} e^{-(r+\phi+\frac{1}{m})(s-t)}\frac{1}{m}\left[(1-q_0)L + q_0 V^g_s - V^u_s\right] ds. \tag{A16}$$

The continuation values of the good and the uninformed types can be solved in closed form

$$\begin{aligned} q_0 V^g_t &= \frac{q_0 c + q_0 \phi R}{r+\phi}(1 - e^{-(r+\phi)(t_g-t)}) + e^{-(r+\phi)(t_g-t)} q_0 V^g_{t_g} \\ V^u_t &= \frac{c + \phi\left[q_0 + (1-q_0)\theta\right]R}{r+\phi}(1 - e^{-(r+\phi)(t_g-t)}) + e^{-(r+\phi)(t_g-t)} V^u_{t_g}. \end{aligned}$$

Thus, we get that

$$\begin{aligned} (1-q_0)L + q_0 V^g_t - V^u_t &= (1-q_0)\left[L - \frac{c+\phi\theta R}{r+\phi}(1 - e^{-(r+\phi)(t_g-t)})\right] + e^{-(r+\phi)(t_g-t)}(q_0 V^g_{t_g} - V^u_{t_g}) \\ &= (1-q_0)\left[L - \frac{c+\phi\theta R}{r+\phi} + e^{-(r+\phi)(t_g-t)}\left(\frac{c+\phi\theta R}{r+\phi} - V^b_{t_g}\right)\right]. \end{aligned}$$

Substituting in equation (A16), we get

$$\begin{aligned} \Gamma_t = \frac{\frac{1}{m}(1-q_0)}{r+\phi+\frac{1}{m}}\left(L - \frac{c+\phi\theta R}{r+\phi}\right)\left(1 - e^{-(r+\phi+\frac{1}{m})(t_b-t)}\right) + \\ (1-q_0)\left(\frac{c+\phi\theta R}{r+\phi} - V^b_{t_g}\right)e^{-(r+\phi)(t_g-t)}\left(1 - e^{-\frac{1}{m}(t_b-t)}\right). \end{aligned}$$

After substituting $V^b_{t_g}$, we get the following equation for t_a:

$$\begin{aligned} \frac{\frac{1}{m}(1-q_0)}{r+\phi+\frac{1}{m}}\left(L - \frac{c+\phi\theta R}{r+\phi}\right)\left(1 - e^{-(r+\phi+\frac{1}{m})(t_b-t_a)}\right) + \\ (1-q_0)\left(\frac{c+\phi\theta R}{r+\phi} - \frac{c+\phi\theta R + \frac{1}{m}\bar{V}^b}{r+\phi+\frac{1}{m}}\right)e^{-(r+\phi)(t_g-t_a)}\left(1 - e^{-\frac{1}{m}(t_b-t_a)}\right) = \frac{\psi}{\lambda}. \end{aligned} \tag{A17}$$

Combining equations (A17) and (A14), together with the incentive compatibility condition determining $t_g - t_b$ in equation (29), we obtain three equations to characterize the thresholds $\{t_a, t_b, t_g\}$:

$$\bar{q} = \frac{1}{1 - \pi^b_{t_a} + \pi^b_{t_a} e^{-\frac{1}{m}(t_b - t_a)}} q_{t_a} \tag{A18a}$$

$$\frac{\psi}{\lambda} = \frac{\frac{1}{m}(1-q_0)}{r+\phi+\frac{1}{m}} \left(L - PV^b_r\right)\left(1 - e^{-(r+\phi+\frac{1}{m})(t_b - t_a)}\right)$$
$$+ (1-q_0)\left(PV^b_r - \frac{c + \phi\theta R + \frac{1}{m}\bar{V}^b}{r+\phi+\frac{1}{m}}\right) e^{-(r+\phi)(t_g - t_a)}\left(1 - e^{-\frac{1}{m}(t_b - t_a)}\right) \tag{A18b}$$

$$t_g = t_b + \frac{1}{r+\phi}\log\left(\frac{V^b_{t_g} - PV^b_r}{L - PV^b_r}\right). \tag{A18c}$$

The final step is to find conditions for an equilibrium with learning (i.e. $t_a > 0$). Let $\bar{t}_a$ be the threshold the first time $q_t = \bar{q}$ in the benchmark model in which $\psi = 0$, which is the same as if $t_a = t_b$. If $t_a = \underline{t}_a \equiv \frac{1}{\lambda}\log\left(\frac{\bar{q}}{1-\bar{q}}\frac{1-q_0}{q_0}\right)$, we have that $\inf\{t > t_a : q_t = \bar{q}\} = \infty$. We have already shown that if $t_a = \bar{t}_a$, then $\Gamma_{t_a} = 0 < \psi/\lambda$. Thus, in any equilibrium $t_a < \bar{t}_a$. It is therefore sufficient to show that if $t_a = \underline{t}_a$, then $\Gamma_{t_a} > \psi/\lambda$. We have that $\lim_{t_a \to \underline{t}_a} t_g = \lim_{t_a \to \underline{t}_a} t_b = \infty$, which means that

$$\frac{\frac{1}{m}(1-q_0)}{r+\phi+\frac{1}{m}} \left(L - PV^b_r\right)\left(1 - e^{-(r+\phi+\frac{1}{m})(t_g - t_a)}\right)$$
$$+(1-q_0)\left(PV^b_r - \frac{c + \phi\theta R + \frac{1}{m}\bar{V}^b}{r+\phi+\frac{1}{m}}\right) e^{-(r+\phi)(t_g - t_a)}\left(1 - e^{-\frac{1}{m}(t_g - t_a)}\right) \to \frac{\frac{1}{m}(1-q_0)}{r+\phi+\frac{1}{m}}\left(L - PV^b_r\right).$$

Hence, there exists a $t_a \in (\underline{t}_a, \bar{t}_a)$ such that $\lambda\Gamma_t = \psi$ if and only if

$$\frac{\psi}{\lambda} < \frac{\frac{1}{m}(1-q_0)}{r+\phi+\frac{1}{m}}\left(L - PV^b_r\right).$$

Finally, we can verify that if the previous condition is not satisfied, then there is no learning in equilibrium. Suppose that the firm never learns and never goes to the market. In this case, the value of the project is

$$V^u = PV^u_r = \frac{c + \phi(q_0 + (1-q_0)\theta)R}{r+\phi},$$

so the value of bank at loan rate y is

$$B^u = \frac{yF + \frac{1}{m}V^u + \phi(q_0 + (1-q_0)\theta)R}{r+\phi+\frac{1}{m}}.$$

Next, suppose that the bank becomes informed (which only occurs off the equilibrium path). In this case, for any loan rate y, the continuation values for the good and bad types are

$$B^b = \frac{y + \frac{1}{m}L + \phi\theta F}{r + \phi + \frac{1}{m}}$$
$$B^g = \frac{y + \frac{1}{m}V^g + \phi F}{r + \phi + \frac{1}{m}},$$

where

$$V^g = PV_r^g.$$

Combining the previous expressions, we get that

$$q_0 B^g + (1 - q_0)B^b - B^u = \frac{\frac{1}{m}(1 - q_0)}{r + \phi + \frac{1}{m}} \left(L - PV_r^b\right),$$

which means that not learning is optimal if

$$\frac{\psi}{\lambda} \geq \frac{\frac{1}{m}(1 - q_0)}{r + \phi + \frac{1}{m}} \left(L - PV_r^b\right).$$

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