A Dynamic Theory of Bank Lending, Firm Entry, and Investment Fluctuations

Yunzhi Hu*

January 25, 2021

Abstract

This paper identifies a two-way, dynamic feedback between bank lending standards and firm entry. The composition of borrowers affects banks' decisions to lend with or without screening and the credit terms they offer. Likewise, bank lending standards affect potential entrepreneurs' decisions to start new businesses, which vary the borrower pool. Firms delay borrowing when they wait for banks to screen or when they expect the borrower pool to improve soon. The model's predictions are consistent with several facts on bank lending and firm entry. Moreover, the results have implications for investment recoveries after bad economic shocks.

Keywords: dynamic adverse selection, real options, dynamic games, lending standards, credit cycle, endogenous entry

*Hu: Kenan-Flagler Business School, UNC Chapel Hill; Yunzhi_Hu@kenan-flagler.unc.edu.

1 Introduction

Banks lend to credible borrowers to make investments that ultimately benefit the real economy. The criteria that banks use to select borrowers, as well as the credit terms they offere, form their lending standards. Apparently, these standards vary with banks' perceived creditworthiness of the borrower pool. Researchers have documented that lending standards are more stringent during economic downturns when delinquencies and defaults are considered more likely (Lown and Morgan, 2006). At the same time, more entrepreneurs choose to start their businesses during economic booms, whereas the quality of these new firms, measured by productivity, survival probabilities, and innovations, are lower than those started during downturns (Lee and Mukoyama, 2015; Moreira, 2015). Given that these new firms are likely credit constrained and bank credit is their most important source of financing (Robb and Robinson, 2012), they form banks' pool of new credit applicants. Naturally, these firms' decisions to borrow should depend on the prevailing lending standards.

In this paper, we develop a theory in which both lending standards and the composition of the borrower pool are determined endogenously, and they interact with each other. Lending standards reflect (i) whether banks screen borrowers and (ii) whether well-qualified borrowers are able to receive quick financing. Lending standards are low when banks provide credit without screening and immediately to all borrowers. By contrast, lending standards are high when banks only offer credit after having carefully screened borrowers or when unscreened credit is not immediately available; that is, borrowers need to wait for it. Naturally, lending standards depend on the composition of borrowers who seek financing. For example, when banks anticipate that many low-productivity borrowers are applying for loans, they tighten the supply of credit.

On the other hand, lending standards also influence a potential entrepreneur's decision to start a new firm, which, given that she/he is likely credit constrained and needs to borrow, is also the decision to enter the borrower pool. Once a potential entrepreneur gets a business idea, she/he needs to take some actions before applying for credit, such as writing up business plans and running pilot experiments. The effort, resources, and expenditures incurred during this process are costly, and whether the potential entrepreneur pursues this idea depends on her/his anticipation of the prevailing lending standards. For example, if credit is immediately available at low interest rates, the payoff from taking these actions to enter the borrower pool is high. Consequently, many potential entrepreneurs enter the pool, and a large fraction of them could have unprofitable projects, contaminating the borrower pool. By contrast, the payoff from entering the pool is low if credit is not available soon, which can happen when banks approve credit only after careful and time-consuming screening. Alternatively, banks may be reluctant to offer quick credit until the perceived borrower pool has improved substantially, because only then can they offer interest rates that are low enough for good borrowers to accept. In any case, potential entrepreneurs' decisions to enter banks' borrower pool are modulated by lending standards.

The paper identifies a unique, two-way, dynamic feedback between equilibrium bank lending and the set of potential entrepreneurs entering the borrower pool. The model unifies the stylized empirical facts on lending standards and firm entry. In addition, the feedback implies firm investments depend on bank lending standards, new firms' entry decisions, and their dynamic interactions. Indeed, the implied patterns of investments can be related to those after bad shocks, such as industry-wide distresses or macroeconomic recessions. Investment recoveries during these episodes are sometimes slow, sometimes fast, and may experience double dips.

Let us be more specific. The model embeds a dynamic adverse selection problem into a banking setting. Borrowers are of either high or low quality, and only high-quality borrowers' investment projects have positive net present value (NPV). All borrowers apply for loans from banks that are ex-ante uninformed of borrowers' types. Banks can either offer unscreened credit (pooling offer) or screen borrowers and provide credit after becoming informed (screened offer). The paper enriches the standard adverse selection model along two dimensions. First, it allows for dynamic information production: banks are endowed with time-consuming screening technology that enables them to learn the quality of borrowers over time. Consequently, banks and firms decide not only the type of offer, but also the time to issue or accept it. Second, the model endogenizes the *average* quality of borrowers through the entry decisions of potential entrepreneurs. To enter the borrower pool, a potential entrepreneur needs to pay an entry cost. The borrower pool improves (deteriorates) over time if the current average quality is below (above) the *marginal* quality, defined as the quality of entrepreneurs who decide to enter.

Upon entry, borrowers are able to apply for bank credit. However, they may wait for a desirable offer and won't borrow until then. Such waiting and the resulting delay in investment can be optimal because bank screening takes time and the average quality of the borrower pool may increase. In the stationary equilibrium whereby the average quality of the borrower pool stays unchanged, high-quality borrowers agree to immediately accept a pooling offer if and only if the interest rate is sufficiently low, which equivalently requires the average quality of the borrower pool to be sufficiently high. Otherwise, they would rather wait for the screened offer. Banks, which do not want to lend to low-quality borrowers, never issue pooling offers unless they expect high-quality borrowers to accept them. If the entry cost is high relative to the surplus of the project, this force discourages low-quality potential entrepreneurs from entering and contaminating the borrower pool, and the equilibrium features all borrowers immediately accepting pooling offers. If, however, the entry cost is low relative to the surplus of the project, the stationary equilibrium cannot accommodate immediate pooling offers. Instead, it features a delay in banks issuing pooling offers. In this case, borrowers need to wait to get their projects financed, and high-quality borrowers may also receive screened credit offers while waiting.

The transition and convergence to the stationary equilibrium may also involve delay and borrowers waiting. Along the transition path, the average quality of the borrower pool may increase over time, which gives high-quality borrowers an additional reason to wait. Indeed, when they expect the borrower pool to improve, which will be the case if the marginal quality exceeds the average quality, they will postpone borrowing, because the interest rate of the pooling offer is expected to decrease over time. If the economy starts with a level of average quality that is far below the level in the stationary equilibrium, the convergence path is characterized by a waiting region and a pooling region. During the waiting region, high-quality borrowers postpone borrowing until after banks have screened them. They will not take immediate pooling offers until the average quality has increased substantially to the level in the pooling region. Therefore, banks only lend after screening, and lending standards are high. The dynamic waiting effect highlights the feedback from firm entry to bank lending. Expecting a delay in receiving credit, low-quality potential entrepreneurs will not enter the borrower pool, and thus, the pool indeed improves. This endogenous entry effect highlights the feedback from equilibrium lending to firm entry.

The two-way feedback has important implications for how investments fluctuate following economic shocks. After a good shock, such as an economic boom, all borrowers accept pooling offers and invest immediately. Many low-quality potential entrepreneurs enter and contaminate the borrower pool. The larger and longer the boom is, the more the average quality drops. Interestingly, after bad shocks, such as permanent or persistent shifts from high to low levels of projects' cash flows, high-quality borrowers postpone their borrowing if the average borrower quality at the onset of the shock is sufficiently low. Therefore, our model predicts investment recoveries are particularly slow after large and long-term economic booms. This pattern is verified using the data across different countries, as well as different industries in the US. By contrast, if average borrower quality at the onset of the shock is relatively high (as in the case of small and short-term booms), the convergence path only involves a pooling region. In this case, banks make pooling offers, and investment recoveries are relatively fast.

Bank screening generates useful information on the borrowers' business prospect but cannot be shared with others. Once screening is endogenized as a costly decision chosen by banks, two additional results are obtained due to the interaction between bank screening and firm entry. First, the market can completely freeze, in which case, no credit is available to any borrower. Lending standards are the highest in this case, because banks issue neither screened nor unscreened credit. Second, following bad economic shocks, investment recovery can be non-monotonic, such as experiencing a double dip. In a double-dip recovery, credit freezes initially, recovers for some time, and then freezes again before it finally recovers. The non-monotonic patterns in credit issuance and the resulting investment are driven by the banks' endogenous level of screening, which varies as the composition of the borrower pool changes due to entry.

This paper is built on the literature on private learning and dynamic adverse selection (Daley and Green, 2012; Kremer and Skrzypacz, 2007; Kaniel and Orlov, 2020; Fuchs and Skrzypacz, 2015; Chang, 2018; Kaya and Kim, 2018), which has established delay and waiting as a core mechanism to separate borrowers. We apply the insights from this literature and show adverse selection dynamics can be related to delays in investment recoveries. Our model, along with Zryumov (2015), introduces endogenous borrowers' entry and emphasizes the feedback between delay/waiting and entry. A key difference between the two papers is the efficiency implications. In Zryumov (2015), low-quality borrowers' projects have positive NPV, so delay is never efficient. By contrast, in our model, low-quality borrowers' projects have negative NPV, so delay/waiting may improve efficiency by discouraging too much entry by low-quality borrowers. Another distinction between this paper and much of the literature on dynamic adverse selection lies behind the "news" process. Whereas the existing literature typically assumes an exogenous and *public* news process, this paper endogenizes the sources of *private* news as banks' screening decisions and also endogenizes banks' efforts in producing news. Due to these endogenous decisions, this paper can generate double dips and episodes during which the market completely collapses. On the more applied side, this paper is among the first to introduce dynamic adverse selection to bank lending (also see Hu and Varas (forthcoming); Halac and Kremer (2020); Lee and Neuhann (2019)).

This paper is also directly related to the theoretical works on banks' lending standards (Daley et al., 2020; Fishman and Parker, 2015; Ruckes, 2004; Dell'Ariccia and Marquez, 2006; Figueroa and Leukhina, 2015; Bolton et al., 2016). Whereas most of this literature is static, our model is dynamic and emphasizes the dynamic and endogenous feedback between bank lending and the composition of the borrower pool. Two recent developments in understanding the dynamics of lending standards are Fishman et al. (2020) and Farboodi and Kondor (2020). Compared with both papers, the mechanism in this paper emphasizes borrowers' delay and waiting in accepting unscreened bank credit. In this sense, our paper is related to the literature on investment as a real option (Dixit et al., 1994). This paper's implications on the credit cycle are also related to Acharya and Viswanathan (2011), who emphasize that booms induce excessive entry by highly-levered firms, which worsens liquidity when downturns hit. More broadly, this paper is related to the literature on adverse selection in the financial market and the macroeconomy (Daley and Green, 2016; Eisfeldt, 2004; Gorton and Ordoñez, 2014; Malherbe, 2014; Kurlat, 2013). We focus on the dynamic and endogenous composition of the borrower pool, which leads to time-varying credit quality and fluctuations in investment patterns.

2 Model

We consider a continuous-time model with an infinite horizon. The economy is populated with three groups of agents, all of whom are risk-neutral and discount the future at a rate ρ. In each period, a pool of heterogeneous *borrowers* apply to a set of competitive *banks*. Banks are endowed with a screening technology and may lend with or without screening. These decisions, as well as the terms of lending, depend crucially on the composition of the borrower pool, which, in turn, is endogenously determined through the entry decisions of *potential entrepreneurs*. Below, we describe the model in detail.

2.1 Borrowers and Projects

Borrowers are penniless, and each has access to a project that requires a fixed amount of investment I. Upon investing, the project generates some cash flows R. One can interpret R either as some instantaneous cash flows or as the discounted present value of all cash flows generated by a long-term project. The project is of heterogeneous quality, modeled as differences in the probability of experiencing a liquidity shock of size ℓ after investment I has been made. Specifically, a high-type project is never hit by the shock, whereas a low-type project is always hit by one. Once the shock hits, the borrower needs funding to defray it; otherwise, the project is liquidated with no scraping value. We assume the size of the shock satisfies $\ell < R$ so that providing the additional liquidity is ex-post (after the initial investment I is made) efficient. However, $I < R < I + l$, so that ex ante (before I is made), only the high-quality project has a positive NPV.

Borrowers should be interpreted as startups and young businesses without well-established credit history yet. We assume each of them is infinitesimal. Let n_t^h and n_t^l be the total measure of high- and low-type borrowers in the pool at time t. $N_t = n_t^h + n_t^l$ is thus the total size of the borrower pool. The state variable in our model is $\mu_t = \frac{n_t^h}{N_t}$, which stands for the pool's *average quality*.

2.2 Potential Entrepreneurs' Entry

μ_t – the average quality – evolves endogenously as potential entrepreneurs enter the pool over time. During a short period $[t, t + dt)$, a flow of potential entrepreneurs are born; that is, they get a business idea. Among them, $\eta N_t dt$ are of high quality, and $\frac{1-q}{q}\eta N_t dt$ are of low quality.[1] Not all

[1]The assumption that potential entry is proportional to the size of the borrower pool is made for tractability. See Appendix A.2.1 for an analysis where potential entry is independent of the size of the pool.

the potential entrepreneurs will enter the borrower pool. The process of entry can be considered the costly actions taken between getting a business idea and transforming this idea into a venture so that one can apply for credit. We focus on how the average quality of the borrower pool μ_t varies with the marginal quality, defined as the quality among potential entrepreneurs who choose to enter. Clearly, the marginal quality depends on the decisions by both the high- and low-type potential entrepreneurs. To simplify the analysis, we fix the entry decisions of high-type potential entrepreneurs by assuming they always enter, whereas low-type potential entrepreneurs need to pay an entry cost c. This cost includes the expenditures incurred during pilot experiments and writing up business plans. Alternatively, one can think of the cost as the outside option of starting a business, such as working as a rank-and-file employee for an existing company. Entry is a one-time decision and is non-recallable, so once a potential entrepreneur forgos the opportunity, she/he will permanently leave the economy. Under endogenous entry by low-type potential entrepreneurs, the marginal quality varies between $\underline{q}$ and 1.

To focus exclusively on the effect of entry, we introduce a replacement assumption. Specifically, whenever a borrower receives financing and exits the borrower pool, she/he will be replaced with a new borrower of identical quality. Under this assumption, the composition of the borrower pool remains unchanged in the absence of entry. This assumption allows us to isolate the effect of entry from exiting; it has also been used in previous literature, such as Tadelis (1999). In subsection 4.3, we show the qualitative results of the model stay unchanged when we drop the replacement assumption and explicitly study borrower exiting. Moreover, we show in that subsection that under the replacement assumption, forming lending relationships is isomorphic to borrowers exiting the pool, and therefore does not change the results either.

2.3 Banks, Screening, and Lending Standards

We model a competitive set of banks, each of which has a perfectly elastic supply of capital. As a result, banks are willing to make offers as long as they expect to break even. During a short period $[t, t + dt)$, a borrower is allowed to apply for screening to one bank.[2] The borrower does not commit to any long-term banking relationship, and is allowed to switch her/his application to a different bank in the next period.[3]

Banks are ex-ante uninformed: they observe neither the type of a specific borrower nor the time she/he entered the pool, but they know the distribution of borrowers, characterized by μ_t. In

[2]Given the assumption of Poisson information arrival introduced below, our results will go through as long as a borrower can apply to at most a finite number of banks.

[3]The absence of relationship lending is consistent with the assumption below that a borrower's history, in particular, the time of entry, is not publicly observable.

equilibrium, μ_t is also the belief that banks assign to a borrower whose type is yet unknown, given that borrowers are allowed to switch to a different bank across periods. Although banks are ex-ante uninformed, they can become ex-post informed. Specifically, each bank is endowed with a screening technology that enables it to learn the true type of a borrower at a Poisson rate s. Note we depart from existing studies on bank screening by assuming information arrival takes time. In reality, loan officers collect soft information on borrowers through frequent and personal contacts, which are time-consuming. Conditional on information arrival, screening produces perfect information; that is, information arrival fully reveals a borrower's type.[4] In the benchmark model, we assume screening does not incur any cost to the bank. Subsection 4.1 studies the case where screening incurs a private cost, and the rate of information arrival s is then chosen endogenously.

If screening generates information, that information is only observable to the specific bank, which naturally generates an information monopoly (Rajan, 1992). This information monopoly inhibits competition, so that the informed bank is able to share some surplus generated from screening. For simplicity, we assume the borrower and the bank bargain (in a Nash manner) over the surplus, with β and $1 - \beta$, respectively, being the relative bargaining power. The bargaining power can be microfounded by alternating offers (Rubinstein and Wolinsky, 1985; Rubinstein, 1982). We assume the bargaining and negotiation between one borrower and her/his bank is not observed by other lenders, consistent with the practice. For the remainder of this paper, we refer to the face value determined by bargaining as a screened offer.

On the other hand, if screening does not generate any information, the bank may still make an uninformed, pooling offer. To capture the competitive nature of the banking sector, we also allow other banks, namely, banks that the borrower hasn't applied to yet, to compete in the terms of the pooling offer. Throughout the paper, offers – both pooling and screened ones – are assumed to be private and observed only by the offering bank and the borrower. In much of the paper, we refer to the case in which pooling offers are immediately made and accepted as periods of low lending standards. By contrast, if pooling offers are not immediately available and high-type borrowers may only be financed with screened offers, we refer to this case as periods of high lending standards.

One should interpret banks in our model as any financial institution that is capable of producing information on borrowers and making loans. They can be commercial banks, community banks, venture capitalists, and credit unions. When banks do not screen, they can also be interpreted as

[4]To be precise, the results are derived in a model where banks have a small probability $\pi \to 0$ of making a type II error; that is, recognizing low-type borrowers as high types. For any $\pi > 0$, low types will apply for screening. When $\pi \equiv 0$, low types are indifferent between applying or not, because no cost for being screened is associated with applying. However, they must apply in equilibrium; otherwise, banks do not need to screen afterwards. Note that in principle, the bank can charge an application fee to separate borrowers. This is infeasible here because all borrowers are penniless.

non-bank financial institutions and the financial market.

2.4 Strategies and Equilibrium

The sequence of events during a short period $[t, t + dt)$ goes as follows. First, banks decide whether to make unscreened pooling offers to borrowers in the pool, and if these offers are made, borrowers choose between accepting and rejecting. Next, each borrower who has not been funded can apply to one bank for screening, and if screening generates information, the informed bank and the borrower bargain over the face value of the screened offer. Finally, potential entrepreneurs are born and decide whether to enter the pool.

Let V_t^h and V_t^l be the continuation value of a high- and low-type borrower at time t, respectively. Let F_t^h be the face value of the screened offer to a high-type borrower at time t. If bargaining fails, the bank and the borrower receive 0 and V_t^h. The surplus of bargaining, $(R - I) - V_t^h$, is then split between the two parties. It becomes immediately clear that

$$F_t^h = (1 - \beta)\left(R - V_t^h\right) + \beta I, \tag{1}$$

where, again, β is the bargaining power of the high-type borrower. On the other hand, if screening generates information that the borrower is of low quality, the bank will not lend, and the borrower will return to the pool. Finally, until screening has generated any information, the borrower is still treated as one whose type is unknown. In this case, she/he may still receive a pooling offer. Due to the assumption of competition, the face value of a pooling offer is straightforward. Let $\hat{\mu}_t$ be the average quality of borrowers conditional on accepting the pooling offer. The face value of the pooling offer F_t^p thus satisfies:

$$F_t^p = I + (1 - \hat{\mu}_t)\,\ell, \tag{2}$$

where $(1 - \hat{\mu}_t)\,\ell$ compensates the bank for potentially funding a low-quality project and lending an additional amount of ℓ after I is made. Define $\mu_{\min}^p = 1 - \frac{R-I}{\ell}$ as the minimum level of $\hat{\mu}_t$ at which a pooling offer is still feasible: if $\hat{\mu}_t < \mu_{\min}^p$, then $F^p(\hat{\mu}_t) > R$. For the remainder of this paper, we focus on the region in which $\mu_t \geq \mu_{\min}^p$. We also suppress the time subscript of the pooling offer and use the function $F^p(\hat{\mu}_t)$. Clearly, $\hat{\mu}_t$ depends on borrowers' equilibrium strategies.

The borrower's problem is straightforward. Given the offer processes F_t^h and $F^p(\hat{\mu}_t)$, a borrower of type $\theta \in \{h, l\}$ chooses a stopping time τ^θ to accept the offer:

$$V_t^\theta = \sup_{\tau^\theta} \mathbb{E}\left[e^{-\rho\tau^\theta}\left(\mathbb{1}_{\tau^\theta \notin T^s}\left(R - F^p\left(\hat{\mu}_{\tau^\theta}\right)\right) + \mathbb{1}_{\tau^\theta \in T^s}\mathbb{1}_{\theta = h}\left(R - F_{\tau^\theta}^h\right)\right)\right], \tag{3}$$

where T^s is the set of (stochastic) time at which a bank learns the borrower's true type. If $\tau^\theta \notin T^s$, the borrower accepts a pooling offer and receives $R - F^p(\hat{\mu}_{\tau^\theta})$. If $\tau^\theta \in T^s$, however, the borrower receives $R - F^h_{\tau^\theta}$ if and only if her/his type is high. Note a low-type borrower's continuation payoff V^l_t is also the time-t entry benefit of a low-type potential entrepreneur. Therefore, she/he enters if and only if the benefit exceeds the cost; that is,

$$V^l_t \geq c. \tag{4}$$

Definition 1. *An equilibrium is a set of processes* $\{\mu_t\}$, $\{F^h_t\}$, *and* $F^p(\hat{\mu}_t)$, *and the borrower's optimal stopping time* $\{\tau^h, \tau^l\}$, *such that*

1. *Optimality:* τ^θ *solves borrowers' problem* (3) *for type* $\theta \in \{h, l\}$. F^h_t *and* $F^p(\hat{\mu}_t)$ *satisfy* (1) *and* (2). *Low-type potential entrepreneurs enter the pool if and only if* (4) *holds.*

2. *Belief Consistency:* μ_t *is consistent with the entry decisions;* $\hat{\mu}_t$ *is consistent with* τ^θ, $\theta \in \{h, l\}$.

3. *No (unrealized) Deals:* $V^h_t \geq R - F^p(\mu_t)$.

Conditions (1) and (2) are standard. Condition (3) says a high-type borrower's continuation value must be at least as high as the value she/he receives by immediately accepting a pooling offer with face value $F^p(\mu_t)$. This condition, widely used in the literature on dynamic adverse selection, can be microfounded by the assumption that offers are private and banks compete in issuing pooling offers.[5] If this condition is violated, a bank can issue a pooling offer in the range of $\left(F_p(\mu_t), R - V^h_t\right)$ to a random borrower in the pool, which earns strictly positive profits in expectation. Condition (3) implies the following results.

Lemma 1. *In any equilibrium, the pooling offer may only be made at* $F^p(\mu_t)$*; that is,* $\hat{\mu}_t \in \{0, \mu_t\}$.

Lemma 1 implies whenever a pooling offer is made and accepted with a positive likelihood, banks expect high and low types to accept it with the same probability. This result will largely simplify the analysis, because it reduces the set of candidate pooling offers at time t to a single choice $F^p(\mu_t)$. Let us offer a heuristic proof. Think about an arbitrary pooling offer. If high types accept with a strictly higher probability, it must be that low types play a mixed strategy. Therefore, a low-type borrower's payoff from accepting the immediate pooling offer is identical to

[5] The condition is introduced as No-Deals in Daley and Green (2012) and market clearing in Fuchs and Skrzypacz (2015). A distinction in this paper is that news is private, so that only pooling offers are subject to competition. Instead, screened offers are determined by bargaining, due to the information advantage of the informed bank relative to other lenders.

one in which she/he accepts some other offer in the future. However, high types will be strictly better-off accepting this same future offer, because she/he may also receive a screened offer between now and then. On the other hand, if high types have a lower probability of accepting the pooling offer $F_t^p(\hat{\mu}_t)$, a pooling offer $F_t^p(\hat{\mu}_t) - \epsilon$ with ϵ sufficiently small will generate strictly positive profits to the bank, and high types will have strict incentives to accept it. This deviation violates the No-Deals condition.

Given Lemma 1, we drop the notation $\hat{\mu}_t$ for the remainder of this paper.

2.5 Discussion of Assumptions and Parametric Restrictions

Borrower Heterogeneity. Note borrower heterogeneity is modeled as different needs for additional liquidity. This modeling choice allows us to obtain closed-form solutions. Implicitly, we assume banks cannot commit to not offering additional funding after the liquidity shock hits. In section 4.2, we eliminate the liquidity shock ℓ, and instead model borrower heterogeneity as differences in the probability of generating the cash flows R. All results will go through.

Moreover, we have implicitly assumed high-type potential entrepreneurs do not need to pay the entry cost c. This assumption is a simplification so that we only need to focus on the one-dimensional entry problem, while still allowing the marginal quality to vary in a wide range. Subsection 4.2 shows the results will go through if both types need to pay the same cost. The crucial assumption is that the NPV of the two types of projects are different, so the benefits of entry (as opposed to the costs of entry) are different.

Bank Competition and Nash Bargaining. We have assumed pooling offers are subject to competition. By contrast, screened offers are not subject to competition, because the information produced from screening is private, which should naturally give surplus to the informed bank. Nash Bargaining is one simple approach to model the division of this surplus.

Perfect Bank Screening. The assumption that bank screening generates perfect information is made without loss of generality. In practice, bank screening can make both type-I and type-II errors. In the case of a type-I error, a good-type borrower returns to the pool and keeps applying. In the case of a type-II error, the bank lends to a low-quality borrower, and the face value of the screened offer adjusts for this mistake. As long as the probability of either type of error is not too high so that screening is useful, the results of this paper will go through.

To focus on the most interesting case, we make a few parametric assumptions.

Assumption 1. $\underline{q}$ *is sufficiently low.*

When $\underline{q}$ is sufficiently low, the size of potential low-type entrepreneurs $\eta\frac{1-\underline{q}}{\underline{q}}N_t dt$ is sufficiently large. Therefore, the idea behind Assumption 1 is that, anyone can come up with a business idea that does not need to have the potential to be developed into a profitable business model. This assumption allows us to study a wide range of the marginal quality, which varies between $\underline{q}$ and 1. Appendix A.1.3 presents the results for any level of $\underline{q}$.

Assumption 2.

$$c < R - I.$$

Assumption 2 requires that the entry cost is not too high, and in particular, that the cost falls below the NPV of a high-quality project. Otherwise, low-type potential entrepreneurs will never enter the borrower pool.

Finally, to simplify the exposition, the results presented in the next section are based on the following assumption.

Assumption 3.

$$\beta = 1.$$

Under Assumption 3, a high-type borrower makes a take-it-or-leave-it offer to the informed bank in Nash bargaining. This assumption implies $F_t^h \equiv I$, so banks do not receive any profit from screening. Subsection 4.1 describes the results for any β, which becomes important when bank screening is endogenized as a costly decision.

3 Equilibrium

In subsection 3.1, we solve for the stationary equilibrium whereby the average quality μ_t stays unchanged over time. Results show a two-way feedback between potential entrepreneurs' entry and the equilibrium bank lending. Subsection 3.2 further studies the convergence path to a stationary equilibrium, which also highlights the dynamic feedback between entry and lending. We relate the results to investment fluctuations.

3.1 Stationary Equilibrium

A stationary equilibrium satisfies all conditions in Definition 1 with an additional requirement that the continuation value $\left\{V_t^h, V_t^l\right\}$ and the average quality μ_t stay unchanged.[6] Let μ^* be the average quality in the stationary equilibrium.

Because low types have negative NPV projects, they may only receive funding by imitating high types. Lemma 1 implies that in a stationary equilibrium, the face value of a pooling offer is always set as $F^p(\mu^*)$. Accepting an immediate pooling offer gives a high-type borrower $R - F^p(\mu^*)$. On the other hand, if high types accept a screened offer, the face value is lower at $F_t^h = I$. The expected payoff associated with waiting for a screened offer is $\frac{s}{\rho+s}\left(R - F^h\right)$, given that screening takes time. Lemma 2 shows accepting an immediate pooling offer is desirable if and only if the average quality μ^* is sufficiently high.

Lemma 2. *Let*

$$\mu_{IC} = 1 - \frac{\rho}{\rho+s}\frac{R-I}{\ell}. \tag{5}$$

High-type borrowers prefer waiting for a screened offer to accepting an immediate pooling offer if $\mu^* < \mu_{IC}$, and vice versa. If $\mu^* = \mu_{IC}$, they are indifferent.

Lemma 1 and 2 imply that in any stationary equilibrium, $\mu^* \geq \mu_{IC}$. Otherwise, no pooling offers are made, because only low types will accept them. Expecting so, low-type potential entrepreneurs will not enter the pool, and the average quality converges to $\mu^* = 1$, which cannot be a stationary equilibrium, because the adverse selection problem no longer exists. The remaining question is whether $\mu^* > \mu_{IC}$ or $\mu^* = \mu_{IC}$, which turns out to depend on the fundamental variables.

First, consider the case in which the fundamentals are strong: $\frac{s}{\rho+s}(R-I) > c$. Intuitively, the left-hand-side payoff is one in which both types of borrowers accept an immediate pooling offer $F^p(\mu_{IC})$ for a payoff $R - [I + (1-\mu_{IC})\,\ell]$. Fundamentals are strong when this payoff exceeds the entry cost c. In this case, pooling offers $F^p(\mu_{IC})$ cannot be immediately issued. The reason is that if they were, all low-type potential entrepreneurs would enter the pool, and the marginal quality would become $\underline{q}$, which is too low to sustain a stationary equilibrium according to Assumption 1.[7] Therefore, the stationary equilibrium must involve pooling offers being issued with a delay. The remaining question is what the face value and the average quality μ^* are. The case $\mu^* > \mu_{IC}$ is ruled out by the No-Deals condition, because banks can make strictly positive profits by issuing

[6] Note the measure of borrowers $\left\{n_t^h, n_t^l\right\}$ may still change. Therefore, the size of the borrower pool N_t may grow or shrink.

[7] In the appendix, we supplement the results for any $\underline{q}$, and indeed, this can be a stationary equilibrium if $\underline{q} > \mu_{IC}$.

pooling offers at face value $F^p(\mu_{IC}) - \epsilon$, where $\epsilon < F^p(\mu_{IC}) - F^p(\mu^*)$, and these pooling offers will be immediately accepted by high-type borrowers. Therefore, $\mu^* = \mu_{IC}$. The stationary conditions require the process of pooling offers to be memoryless; that is, the offers arrive at some Poisson rate. Let ϕ be this rate, which is determined by the condition that low-type potential entrepreneurs are indifferent between entering or not. In equilibrium, exactly $\eta N_t \frac{1-\mu_{IC}}{\mu_{IC}} dt$ of them choose to enter, so the marginal quality stays at μ_{IC}. Upon entry, borrowers are financed with a stochastic delay with expected time $\frac{1}{\phi}$.[8]

Next, consider the case in which the fundamentals are weak: $\frac{s}{\rho+s}(R-I) < c$. Here, low-type potential entrepreneurs will not enter the borrower pool even if they expect to be immediately financed at a pooling offer $F^p(\mu_{IC})$. Therefore, the average quality satisfies $\mu^* > \mu_{IC}$. Specifically, define

$$\mu_c = 1 - \frac{R-I-c}{\ell}. \tag{6}$$

A low-type potential entrepreneur will be indifferent between entering or not if she/he expects to receive an immediate pooling offer $F^p(\mu_c)$. In equilibrium, exactly $\eta \frac{1-\mu_c}{\mu_c} N_t dt$ of them choose to enter, so the average quality stays at $\mu^* = \mu_c$.

Proposition 1 summarizes the above discussions and describes the equilibrium.

Proposition 1 (Stationary Equilibrium)**.** *A stationary equilibrium exists, and the equilibrium is unique unless* $\frac{s}{\rho+s}(R-I) = c$.

1. *Strong fundamentals: if* $\frac{s}{\rho+s}(R-I) > c$, $\mu^* = \mu_{IC}$. *Both types of borrowers are financed by pooling offers* $F^p(\mu_{IC})$ *after a Poisson event that arrives at rate* ϕ, *where* $\frac{\phi}{\rho+\phi}[R - F^p(\mu_{IC})] = c$. *High-type borrowers are also financed by screened offers* $F^h = I$ *at rate* s. *Among low-type potential entrepreneurs,* $\eta \frac{1-\mu_{IC}}{\mu_{IC}} N_t dt$ *of them enter during* $\big[t, t+dt\big)$.

2. *Weak fundamentals: if* $\frac{s}{\rho+s}(R-I) < c$, $\mu^* = \mu_c$. *Both types of borrowers are immediately financed by pooling offers* $F^p(\mu_c)$. *Among low-type potential entrepreneurs,* $\eta \frac{1-\mu_c}{\mu_c} N_t dt$ *of them enter during* $\big[t, t+dt\big)$.

If $\frac{s}{\rho+s}(R-I) = c$, a continuum of stationary equilibrium exists. Given this scenario is a knife-edge case, we focus on the parameter values such that $\frac{s}{\rho+s}(R-I) \neq c$ for the remainder of this paper. Proposition 1 is the first main result. It highlights a two-way feedback between the potential entrepreneurs' entry and bank lending. To see this, let us explore the case of weak and strong economic fundamentals, respectively. In the strong-fundamental case, the project's NPV is

[8] Note high types' payoff is $V^h(\mu_{IC}) = \frac{\phi}{\rho+\phi+s}[R - F^p(\mu_{IC})] + \frac{s}{\rho+\phi+s}(R-I) = \frac{s}{\rho+s}(R-I) = R - F^p(\mu_{IC})$, so the No-Deals condition holds.

high relative to the entry cost, so many low-quality potential entrepreneurs would want to enter. To discourage excessive entry, unscreened pooling offers cannot be immediately issued. Instead, they must be delayed in order to reduce the benefits upon entry. Note that in this case, high-type borrowers are financed by both pooling and screened offers, with a delay, at different interest rates. By contrast, in the weak-fundamental case, the project's NPV is relatively low, so the entry cost itself is enough to discourage excessive entry. Hence, the average quality of the borrower pool stays high and the degree of adverse selection is relatively low. In this case, pooling offers without any delay can be issued. An exogenous increase in cost c; that is, a comparison between the two cases, shows the feedback from firm entry to bank lending.

Interestingly, the equilibrium lending decisions also feed back into the entry decisions, which is immediately clear from the low types' payoff in the two cases of Proposition 1. When pooling offers are immediately issued, they attracts potential entrepreneurs to enter if the face value is sufficiently low. To prohibit excessive entry, pooling offers could be made with a delay as in Case 1; otherwise, the entry cost needs to be sufficiently high to offset the benefits as in Case 2.

The two cases in Proposition 1 imply the stationary equilibrium is never efficient. Specifically, it illustrates an interesting tradeoff between delay and entry. Note the first-best efficient benchmark features all high-type borrowers being immediately financed, whereas low-type borrowers should never receive financing. A delay in high types receiving credit therefore reduces efficiency, because investments of positive-NPV projects are postponed. On the other hand, a delay in receiving credit also reduces the low types' entry benefits, so fewer of them choose to enter. Whereas a longer delay reduces the efficiency at the investment margin by high types, it increases the efficiency at the entry margin by low types. In this sense, our results imply delay in bank financing can sometimes be welfare-improving because it eliminates excessive and inefficient entry. This implication differs from much of the literature on dynamic adverse selection, where low-types' projects still have positive NPV (or, equivalently, "gains from trade" exist between lender and borrower).

Proposition 1 predicts that when pooling offers are made with a delay, screened offers are also made in equilibrium. Empirically, during periods in which loan approval is slow, one should observe more dispersion across the interest rates of bank loans, after controlling for observable characteristics. By contrast, when pooling offers are immediately made and accepted, screened offers are not issued. Empirically, during periods in which loan approval is fast, one should observe less dispersion across the interest rates of bank loans, after controlling for observable characteristics. Cross-sectionally, a comparison between these two cases implies that in industries with more entry barriers such as those with more concentration and/or under more stringent regulation, equilibrium bank loans are issued faster, and the interest rates among borrowers are more homogeneous.

Remark 1. *In the stationary equilibrium as well as the transition path described in the next sub-*

section, a delay sometimes occurs during which borrowers wait. This waiting should be interpreted as market timing in borrowing bank loans by high-type borrowers. Specifically, after setting up the business venture (entry), a high-type borrower could approach banks all the time. Although the borrower keeps contacting the bank for an update of credit conditions, she/he may not borrow immediately. Instead, the borrower waits for the credit terms to improve. As shown in this subsection, the credit terms may improve as the bank screens the borrower and offers lower interest rates. The next subsection introduces another reason the credit terms may improve: more and more high-type borrowers enter the pool.

3.2 Convergence Path and Implications on Investment Recoveries

In the stationary equilibrium, borrowers experience delay in receiving funding if fundamentals are strong. This subsection studies the transition and convergence path toward the weak-fundamental equilibrium (Case 2 in Proposition 1), where borrowers do not experience any delay once the stationary equilibrium is attained. We show that along the convergence path, delay is still possible.

Suppose the economy starts with a borrower pool with the average quality μ_0. We assume $\mu_0 < \mu_c$, which can be microfounded by a permanent downward shift in the project's cash flow R at time 0, so the average quality prior to $t = 0$ is too low compared with the level in the stationary equilibrium.[9] A lower μ_0 can be associated with a larger and longer economic boom during the period prior to $t = 0$, as we show at the end of this subsection.

A first result is low-type potential entrepreneurs will never enter the pool before μ_t reaches μ_c. Intuitively, along the convergence path, a low-type borrower's payoff can never exceed the one of accepting an immediate pooling offer, which falls below $R - [I + (1 - \mu_c)\,\ell] = c$.

Lemma 3. $V_t^l(\mu_t) < c$ *for any* $\mu_t < \mu_c$*.*

Given Lemma 3, μ_t evolves according to the following process:

$$d\mu_t = \eta\,(1 - \mu_t)\,dt > 0, \tag{7}$$

such that the average quality of the borrower pool improves over time. An improving borrower pool introduces an additional reason for high-type borrowers to wait to accept pooling offers. By waiting, they can receive a better pooling offer in the future as the average quality increases. Let $V^h(\mu) = V_t^h(\mu_t)$ for $\mu = \mu_t$. In the region where a high-type borrower waits, the following

[9] Results are similar if the downward shift is persistent and the persistence is sufficiently high. An alternative interpretation for $\mu_0 < \mu_c$ is a large inflow of low-quality potential entrepreneurs at $t = 0$.

Hamilton-Jacobi-Bellman (HJB) equations can be derived by considering her/his continuation value over a short time interval $\left[t, t+dt\right)$:

$$\rho V^h(\mu) = \frac{dV^h(\mu)}{d\mu}\eta(1-\mu) + s\left[(R-I) - V^h(\mu)\right]. \tag{8}$$

The right-hand side of equation (8) describes the benefits of waiting. The first term comes from changes in average quality μ_t induced by firm entry in (7). The second term derives from bank screening, which accrues the total surplus from screening to the high-type borrower given Assumption 3.

As μ_t increases, the incentive to wait decreases, for two reasons. First, the surplus from being verified as a high type $\left[(R-I) - V_t^h\right]$ decreases because the alternative option–accepting an immediate pooling offer–becomes more attractive. Second, the marginal improvement in the borrower pool $\eta(1-\mu_t)$ becomes lower as μ_t increases. Intuitively, the marginal improvement in μ_t needs to (eventually) decrease as μ_t increases; otherwise, the process of μ_t will explode. Because the incentive to wait decreases with μ_t, a threshold $\tilde{\mu}_{IC}$ exists such that high-type borrowers prefer waiting if $\mu_t < \tilde{\mu}_{IC}$ but choose to accept a pooling offer $F^p(\mu_t)$ if $\mu_t > \tilde{\mu}_{IC}$. Specifically, $\tilde{\mu}_{IC}$ is determined by two boundary conditions. First, the value-matching condition $V^h(\tilde{\mu}_{IC}) = R - [I + (1-\tilde{\mu}_{IC})\ell]$ holds so that the borrower is indifferent between waiting and accepting an immediate pooling offer at $\mu_t = \tilde{\mu}_{IC}$. Second, the fact that $\tilde{\mu}_{IC}$ is optimally chosen by high-type borrowers, combined with the No-Deals condition, implies the smooth-pasting condition $\frac{dV^h(\tilde{\mu}_{IC})}{d\mu_t} = \frac{d[R-F^p(\mu_t)]}{d\mu_t} = \ell$.

Lemma 4. *The smooth-pasting condition holds at $\tilde{\mu}_{IC}$. A unique*

$$\tilde{\mu}_{IC} = 1 - \frac{\rho}{\rho+s+\eta}\frac{R-I}{\ell}, \tag{9}$$

satisfies both the value-matching and smooth-pasting conditions.

A comparison between $\tilde{\mu}_{IC}$ and μ_{IC} in (5) shows $\tilde{\mu}_{IC} > \mu_{IC}$ as long as $\eta > 0$. This comparison highlights the new and dynamic reason high-type borrowers choose to wait and is consistent with the cleansing effect of recessions. Intuitively, when high types expect the borrower pool to improve over time, they have additional reasons to wait instead of accepting an immediate pooling offer. As a result, the threshold of waiting is even higher than μ_{IC}, the threshold if the average quality stays unchanged.

Proposition 2. *An equilibrium convergence path toward the stationary equilibrium $\mu^* = \mu_c$ exists, characterized by two regions.*

1. *Waiting region:* $\mu_t < \min\{\tilde{\mu}_{IC}, \mu_c\}$. *High-quality borrowers wait and are only financed with screened offers. Low-quality borrowers do not get financed.*

2. *Pooling region:* $\mu_t \in [\min\{\tilde{\mu}_{IC}, \mu_c\}, \mu_c]$. *All borrowers immediately accept pooling offers.*

Along the convergence path, low-quality potential entrepreneurs do not enter the pool before μ_t *reaches* μ_c.

Proposition 2 includes two cases. The first case is $\tilde{\mu}_{IC} < \mu_c$, illustrated in Figure 1. This case holds if and only if $\eta < \frac{(\rho+s)c - s(R-I)}{R-I-c}$, interpreted as a condition of slow entry. The second case is $\tilde{\mu}_{IC} \geq \mu_c$, and the pooling region shrinks to a singular point μ_c. In both cases, a waiting region exists if μ_0 falls below $\min\{\tilde{\mu}_{IC}, \mu_c\}$. In this region, high types experience delay in getting their projects invested: they only get financed after being screened. Low types, on the other hand, cannot receive any credit. Lending standards are high. In the pooling region, financing is no longer delayed. Lending standards are low.

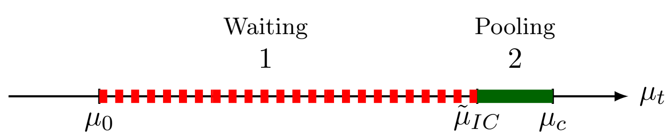

Figure 1: Convergence Path under Exogeneous Screening

Figure 2 plots a high-type borrower's value function. The blue-solid and red-dashed line respectively show the continuation value from waiting until $\tilde{\mu}_{IC}$ and accepting an immediate pooling offer $F^p(\mu_t)$. The vertical line marks the position of $\tilde{\mu}_{IC}$. Clearly, waiting is optimal for a high-quality borrower if and only $\mu_t < \tilde{\mu}_{IC}$.

Proposition 2 highlights a dynamic feedback between entry and lending: when borrowers expect the pool to improve due to entry, they have additional reasons to wait rather than accept an immediate pooling offer. Expecting so, banks will not issue pooling offers, and the payoff upon entering the borrower pool is thus low for a low-type potential entrepreneur. As a result, they choose not to enter, and the resulting borrower pool improves over time.

Proposition 2 has implications for investment fluctuations and recoveries following bad economic shocks at either the industry or macroeconomic level. In particular, it implies investment recovery – measured by the size of total investments – depends on the average quality right after the bad shock hits. If $\mu_0 < \tilde{\mu}_{IC}$ so that the average quality right after the shock is low, recovery is slow. Otherwise, the recovery is fast. To the extent that the average quality μ_0 could depend on the economic conditions prior to the shock and in particular the size and duration of the economic

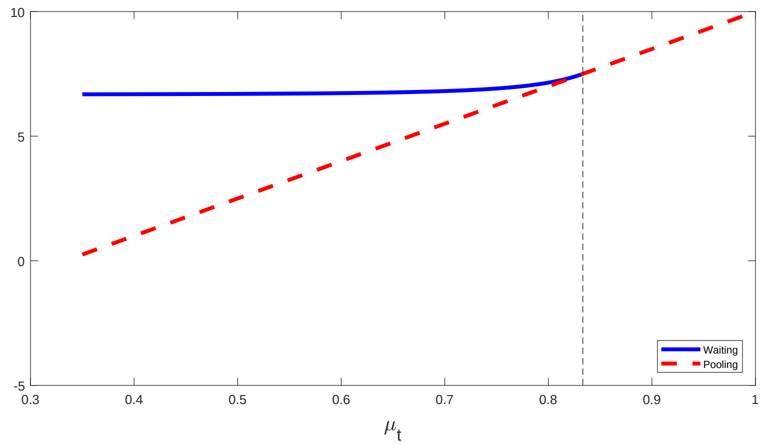

Figure 2: High Types' Value Functions

This figure shows high–quality borrowers' value functions. The horizontal axes represent μ_t–average borrower quality of the borrower pool. The blue curve depicts V_t^h. The red, dashed curve plots the continuation value from immediately accepting a pooling offer at face value $F^p(\mu_t)$. The black-dashed vertical line marks the position of $\tilde{\mu}_{IC}$, the cutoff value at which a high-quality borrower is indifferent between waiting and taking a pooling offer. The parameter values are $I = 20$, $R = 30$, $\ell = 15$, $c = 9$, $\beta = 1$, $s = 1$, $\rho = 0.5$, $\eta = 0.5$, and $\underline{q} = 0.1$. In this case, $\tilde{\mu}_{IC} = 0.83$ and $\mu_c = 0.93$.

boom,[10] this result implies investment recoveries following long-term and large economic booms are particularly slow. During the earlier stage of recovery, only screened offers are made, and credit quality among projects undertaken is high. Even high types cannot (and they choose not to) get their projects financed immediately. Investment is thus delayed, and output falls dramatically. By contrast, following a short and/or small boom, the average quality right after the shock μ_0 may still exceed $\tilde{\mu}_{IC}$. After the same bad shock hits, pooling offers continue to be made. In this case, no investment is delayed.

The dynamics of economic surplus (i.e., output net of investment) has two stages in a slow recovery. Within a short period $[t, t + dt)$, the surplus is $sN_t\mu_t(R - I)\,dt$ when $\mu_t \in (\mu_0, \tilde{\mu}_{IC})$, where N_t is the total measure of borrowers in the pool. After the average quality μ_t improves above $\tilde{\mu}_{IC}$, the surplus becomes $N_t[R - I - (1 - \mu_t)\ell]$. Note that due to the property of immediate pooling offers, the surplus is of higher order in the second stage, but it involves financing low-quality borrowers.

[10] Povel et al. (2007) and Khanna et al. (2008) also provide mechanisms whereby credit quality declines in booms. Also see Acharya and Viswanathan (2011) and Diamond et al. (2020).

The comparison between slow and fast recovery show credit quality and quantity are closely intertwined. When the quality of the borrower pool is low, high-quality borrowers would rather wait for the pool to improve, and therefore, equilibrium credit quantity is low. However, among borrowers who are able to get financed (projects that are undertaken), the credit quality is very high and far exceeds the quality of the borrower pool (potential projects).

Even though the duration of recovery depends on both the rate of borrower entry η and the speed of screening s, the dependence is different. An increase in the entry rate η has two effects. On one hand, (7) shows it leads to a faster improvement of the borrower pool, which, ceteris paribus, accelerates investment recovery. On the other hand, it increases $\tilde{\mu}_{IC}$, because high-type borrowers' incentives to wait are higher when they expect the pool to improve faster. By contrast, an increase in s increases $\tilde{\mu}_{IC}$ and always delays investment recovery. Intuitively, an acceleration in the screening technology reduces the expected waiting time to receive a screened offer, so high types' incentives to wait become higher. This result is a crucial intermediate step to understand the non-monotonic patterns in investment recoveries presented in subsection 4.1.

Finally, this subsection has focused on the convergence to μ_c from μ_0 below. Let us present the results of the convergence from above; that is, if $\mu_0 > \mu_c$. One interpretation is that a permanent (or highly-persistent) good shock such as an economic boom hits at time 0. In this case, the average quality at time 0 exceeds the level in the new stationary equilibrium. The transition is unique and features pooling offers being immediately made at $F^p(\mu_t)$ along the convergence path. Entry benefits satisfy $V_t^l = R - F^p(\mu_t) > c$ so that all low-type potential entrepreneurs enter the pool. Over time, the average quality decreases according to $d\mu_t = \left(1 - \frac{\mu_t}{\underline{q}}\right) dt < 0$ until μ_t reaches μ_c. Intuitively, high-quality borrowers have even lower incentives to wait, when they expect the pool to deteriorate. The convergence path following $\mu_0 > \mu_c$ therefore shows that during an economic boom, the average quality of the borrower declines over time. The longer and larger the boom is, the lower the average quality becomes.

The comparison between two convergence cases shows lending standards are counter-cyclical. Following a good shock, pooling offers are immediately issued and accepted. Lending standards are low, and the average quality declines as the good shock persists.[11] Following a bad shock, Proposition 2 shows a waiting period could exist, during which only screened offers are issued. Lending standards are high, and the average quality improves as the bad shock persists.

Remark 2. *To highlight the role of endogenous firm entry, we now describe the results under*

[11]If the shock is good enough that the new stationary equilibrium has strong economic fundamentals, as in Case 1 of Proposition 1, pooling offers are issued with a delay in the stationary equilibrium once μ_{IC} is attained. However, this delay will only occur after the bad shock has persisted for a long time. Before μ_t reaches μ_{IC}, pooling offers are still issued immediately, and lending standards are low.

exogenous entry, i.e., the marginal quality of entry is fixed at some level $q_\dagger$. If $q_\dagger < \mu_{IC}$, pooling offers will never be made. Only high types receive financing after being screened, and the average quality converges to 0. If $q_\dagger > \mu_{IC}$, a convergence path similar to that in Proposition 2 exists, although the boundary is different from $\tilde{\mu}_{IC}$. Therefore, an important role of endogenous entry is to generate (instead of assume) high marginal quality.

4 Extension, Robustness, and Empirical Implications

4.1 Bargaining Power and Endogenous Screening

This subsection introduces two modifications. First, we describe the equilibrium under interior bargaining power $\beta \in (0,1)$. With bargaining power, the bank, once informed of the borrower's type being high, will earn strictly positive profits due to its information monopoly. Second, we endogenize the level of bank screening (and the Poisson arrival rate of the information) as a costly decision. A new result is that, changes in bank screening introduce another reason for high-type borrowers to wait along the convergence path studied in subsection 3.2. Consequently, lending standards may vary non-monotonically with μ_t, and investment recovery could experience double dips. In a double-dip recovery, credit freezes initially, recovers for some time, and then freezes again before it finally recovers.

Interior Bargaining Power

Under interior bargaining power, the face value of a screened offer is described by (1), which depends on high types' equilibrium payoff. Proposition 3 describes the results.

Proposition 3. *A stationary equilibrium exists, characterized by μ_{IC}^{β} and μ_c.*

1. *Strong fundamentals: if $\frac{s\beta}{\rho+s\beta}(R-I) > c$, $\mu^* = \mu_{IC}^{\beta}$. Both types of borrowers are financed by pooling offers $F^p\left(\mu_{IC}^{\beta}\right)$ after a Poisson event that arrives at rate ϕ^{β}. High-type borrowers are also financed by screened offers (1) at rate s. Among low-type potential entrepreneurs, $\eta\frac{1-\mu_{IC}^{\beta}}{\mu_{IC}^{\beta}}N_t dt$ of them enter during $\left[t, t+dt\right)$.*

2. *Weak fundamentals: if $\frac{s\beta}{\rho+s\beta}(R-I) < c$, $\mu^* = \mu_c$. Both types of borrowers are immediately financed by pooling offers $F^p(\mu_c)$. Among low-type potential entrepreneurs, $\eta\frac{1-\mu_c}{\mu_c}N_t dt$ of them enter during $\left[t, t+dt\right)$.*

3. *The convergence to a weak-fundamental equilibrium is characterized by a waiting region $\left[\mu_0, \min\left\{\tilde{\mu}_{IC}^{\beta}, \mu_c\right\}\right)$ and a pooling region $\left[\min\left\{\tilde{\mu}_{IC}^{\beta}, \mu_c\right\}, \mu_c\right]$.*

4. Both μ_{IC}^{β} and $\tilde{\mu}_{IC}^{\beta}$ increase with β.

The stationary equilibrium and convergence path are similar to those in Propositions 1 and 2. The detailed expressions for μ_{IC}^{β} and $\tilde{\mu}_{IC}^{\beta}$ are in the appendix, and both increase with β. Intuitively, an increase in β reduces the bank's bargaining power, and therefore the face value F_t^h, so that the screened offer is more desirable to a high-type borrower. Ceteris paribus, she/he has higher incentives to wait for it.

Endogenous Screening

To make the results directly comparable to those in subsection 3.2, we focus on the weak-fundamental case $\frac{s\beta}{\rho+s\beta}(R-I) < c$ in Proposition 3 for the remainder of this subsection, where the stationary equilibrium features $\mu^* = \mu_c$.

We next endogenize bank screening. Given the sequence of events, the bank may only screen when pooling offers are not immediately issued in equilibrium. We model screening as a binary-effort choice.[12] At any time t, each bank chooses its effort of screening, $s_t \in \{0, s\}$. Equivalently, private news arrives with approximately probability $s_t dt$ during $\left[t, t+dt\right)$. In other words, the probability that screening generates information depends on the bank's effort at time t but not any other point in time. Whereas no screening $s_t = 0$ incurs no cost, $s_t = s$ incurs a constant flow cost κdt. We require κ to be neither too high nor too low.[13] Once again, Lemma 3 holds such that along the transition path, $d\mu_t = \eta(1-\mu_t)$ for any $\mu_t < \mu_c$.

Proposition 3 shows high-type borrowers' decision between waiting and immediate pooling is characterized by a threshold in average quality $\tilde{\mu}_{IC}^{\beta} = 1 - \frac{\rho}{\rho+s\beta+\eta}\frac{R-I}{\eta}$. Following the similar analysis, define

$$\underaccent{\tilde}{\mu}_{IC} = 1 - \frac{\rho}{\rho+\eta}\frac{R-I}{\eta} \tag{10}$$

as the critical threshold between waiting and immediate pooling if banks never screen; that is, $s_t \equiv 0$, $\forall t$. It is immediately clear that $\underaccent{\tilde}{\mu}_{IC} < \tilde{\mu}_{IC}^{\beta}$. Intuitively, high-type borrowers have higher incentives to wait if they know banks are screening $s_t = s$, because the option value of waiting becomes higher.

Next, turn to the bank's problem. If pooling offers are not immediately made and accepted, the face value of the screened offer (1) implies the expected profit from screening is

$$\Pi(\mu_t) = \mu_t(1-\beta)\left[(R-I) - V_t^h\right], \tag{11}$$

[12] The results can be easily extended to $s_t \in [0, s]$ with a linear cost κs_t.

[13] The detailed expressions are in equation (24) of Appendix A.1.8.

where $R-I$ is the NPV of a high type's project, and V_t^h is the value of a high type's outside option. Intuitively, the informed bank extracts a fraction $(1-\beta)$ of the surplus from a successful screening, which occurs with probability μ_t. With the complementary probability $1-\mu_t$, the borrower is a low type and will not be financed. Clearly, bank profits are low when the average quality μ_t is low. The reason is that when the market is filled with low-type borrowers, the borrower being screened is likely to be of low quality, and the bank cannot make any profit from her/him. In this case, finding a high-type borrower is similar to looking for a needle in a haystack. Therefore, banks do not screen, and $s_t = 0$ for low levels of μ_t. Banks will only switch to screening when μ_t increases such that the possibility of screening a high type increases. A threshold μ_s, which depends on the screening cost κ, captures the switch in screening. If κ is neither too high nor too low, $\mu_s \in \left(\underaccent{\tilde}{\mu}_{IC}, \tilde{\mu}_{IC}^{\beta}\right)$. Prior to μ_t reaching μ_s, high-type borrowers have an additional reason to wait: they would like to wait for the bank to switch from no screening $s_t = 0$ to screening $s_t = s$. Another threshold μ_{dd} below μ_s captures this effect. The equilibrium convergence path is summarized below.

Proposition 4. *Suppose $\eta^2 > s\left(\rho-\eta\right)$ and the screening cost satisfies (24) in the appendix. A quadruple $\left\{\underaccent{\tilde}{\mu}_{IC}, \mu_{dd}, \mu_s, \tilde{\mu}_{IC}^{\beta}\right\}$ exists.*

1. *First dip. For $\mu_t \in \left[\mu_0, \underaccent{\tilde}{\mu}_{IC}\right]$, banks do not screen, $s_t = 0$. No borrowers can receive financing.*
2. *First rise. For $\mu_t \in \left[\underaccent{\tilde}{\mu}_{IC}, \mu_{dd}\right]$, banks do not screen, $s_t = 0$. All borrowers immediately accept pooling offers $F_p\left(\mu_t\right)$.*
3. *Second dip.*
   (a) *For $\mu_t \in \left[\mu_{dd}, \mu_s\right]$, banks do not screen, $s_t = 0$. No borrowers can receive financing.*
   (b) *For $\mu_t \in \left[\mu_s, \tilde{\mu}_{IC}^{\beta}\right]$, banks screen, $s_t = s$. High-quality borrowers wait and are only financed with screened offers. Low-quality borrowers do not receive financing.*
4. *Second rise. For $\mu_t \in \left[\tilde{\mu}_{IC}^{\beta}, \mu_c\right]$, banks do not screen, $s_t = 0$. All borrowers immediately accept pooling offers $F_p\left(\mu_t\right)$.*

Note that under $\kappa = 0$, where screening is costless, the bank always screens $s_t = s^*$. In this case, regions 1-3 in the above proposition combine into one waiting region, and the equilibrium is described by Proposition 3. On the other hand, if $\beta \equiv 1$ such that the bank has no bargaining power, and therefore earns zero profit from screening, any positive screening cost $\kappa > 0$ will lead to a result that the bank never screens; that is, $s_t \equiv 0$. In this case, the equilibrium is similar to that described by Proposition 2, except the boundary between the two regions becomes $\underaccent{\tilde}{\mu}_{IC}$.

Lending standards and investment recoveries are different from those in subsection 3.2. In the first dip, no loan is made, and the lending market completely collapses. Investment is stagnant, and lending standards are the highest. Regions 2 and 3(a) are new equilibrium patterns induced by endogenous and time-varying screening. In region 2, high types would have to wait a long time if they wanted banks to screen them (μ_t is far from μ_s). Therefore, they would rather take pooling offers, which will lead banks to issue them in equilibrium. In this case, lending standards are low. Investments are high and recover temporarily. In region 3(a), however, high types would only need to wait a short time for banks to screen (μ_t is close to μ_s), so they will find it optimal to do so. As a result, no loan is made, and the lending market collapses again. Investment is stagnant, and lending standards return to being the highest. In region 3(b), only high types are financed with screened offers. Investment is delayed (but not stagnant), and lending standards are relatively high. Finally, in the second rise, investment is not delayed, and lending standards are low. Figure 3 offers a graphical illustration.

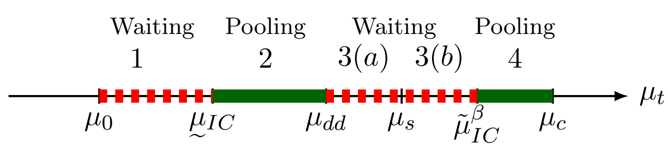

Figure 3: Convergence Path under Endogenous Screening

The dynamics of economic surplus (i.e., output net of investment) is also non-monotonic within a short period $[t, t+dt)$. In the first dip $\mu_t \in \left[\mu_0, \underset{\sim}{\mu}_{IC}\right]$, the surplus is 0, given no investment is ever made. The first rise $\mu_t \in \left[\underset{\sim}{\mu}_{IC}, \mu_{dd}\right]$ has a surplus $N_t\left[R - I - (1-\mu_t)\,\ell\right]$, given that all borrowers are financed immediately with pooling offers. During the second dip, the surplus first goes back to 0 when $\mu_t \in [\mu_{dd}, \mu_s]$, followed by $sN_t\mu_t\,(R-I)\,dt$ when the bank starts to screen during $\mu_t \in \left[\mu_s, \tilde{\mu}_{IC}^{\beta}\right]$. Finally, in the second rise where $\mu_t \in \left[\tilde{\mu}_{IC}^{\beta}, \mu_c\right]$, the surplus becomes $N_t\left[R - I - (1-\mu_t)\,\ell\right]$, which is the highest.

Remark 3. *Note that banks may choose not to screen ($s_t = 0$) when the average quality μ_t becomes very high. The reason is that when μ_t is sufficiently high, V_t^h is also very high. Therefore, $(1-\beta)\left[(R-I) - V_t^h\right]$, the profit that an informed bank can earn from a high type is low. Condition (23) in the appendix requires that the bank screens at $\tilde{\mu}_{IC}^{\beta}$ given the cost κ. Thus, the switch to no screening will occur at some level above $\tilde{\mu}_{IC}^{\beta}$. If (23) is violated, and the switch occurs at some level below $\tilde{\mu}_{IC}^{\beta}$, equilibrium region 2 may cease to exist; that is, regions 1, 2, and 3(a) will combine into one waiting region.*

The general lessons from this exercise are twofold. First, under endogenous screening, the lending market may completely freeze, and neither unscreened nor screened credit exists. Lending standards are the highest during this period. Second, endogenous screening creates an additional reason for borrowers to wait; therefore, recoveries in investment can be endogenously non-monotonic and experience double dips.

4.2 An Alternative Approach to Model Borrower Heterogeneity

In this subsection, we relax two assumptions in the benchmark model of section 2. First, all potential entrepreneurs – both the high and low types – need to pay the cost c to enter the pool. The goal is to show the heterogeneous benefits of entry, as opposed to the cost, lead to an endogenously time-varying borrower pool. Second, we eliminate the liquidity shock ℓ and instead assume cash flow R is only produced with probability θ^h and θ^l, by a high- and low-type borrower, respectively. Therefore, even if banks could commit to no additional lending after the investment I is made, they would not be able to separate the borrowers. We continue to assume only high types' projects have positive NPV; that is, $\theta^h R > I > \theta^l R$.

Because outcomes are binary (R and 0) and borrowers have limited liability, equilibrium contracts can be implemented via debt without loss of generality. Due to the No-Deals condition, Lemma 1 continues to hold, so that the face value of a pooling offer is

$$F^p(\mu_t) = \frac{I}{\mu_t \theta^h + (1-\mu_t)\,\theta^l}, \tag{12}$$

where $\mu_t \theta^h + (1-\mu_t)\,\theta^l$ is the expected probability of producing cash flow R. Compared with (2), $F^p(\mu_t)$ in (12) has the same property of decreasing with μ_t: a higher average quality reduces the face value of the pooling offer. However, in (12), $F^p(\mu_t)$ is non-linear in μ_t, which is the main obstacle that prohibits us from obtaining closed-form solutions. Indeed, it is for this reason that we adopt the liquidity-shock approach in the benchmark model. Note that in this alternative model, high-type borrowers always have a higher payoff than low-type borrowers. The reason is straightforward. By accepting the same pooling offer immediately, high-type borrowers receive $\theta^h (R - F^p(\mu_t))$, which is strictly higher than $\theta^l (R - F^p(\mu_t))$, the payoff received by low-type borrowers.

With some slight abuse of notation, define μ_c as the solution to $\theta^l (R - F^p(\mu_t)) = c$ and μ_{IC} as the solution to $\theta^h (R - F^p(\mu_t)) = \frac{s}{\rho+s}\left(\theta^h R - I\right)$. The stationary equilibrium is unchanged from that in Proposition 1 and therefore omitted. Next, turn to the convergence path to the weak-fundamental stationary equilibrium $\mu^* = \mu_c$. A sufficient condition that guarantees high-type potential entrepreneurs will always enter is $\theta^h (R - F^p(\mu_0)) > c$, where $\mu_0 < \mu^*$ is the average quality at $t = 0$. Equivalently, this condition requires θ^h to be sufficiently high. Under this condition, Lemma 3 continues to hold such that the borrower pool keeps improving until μ_t reaches μ_c. Similar to Proposition 2, the convergence path is characterized by a waiting region $[\mu_0, \tilde{\mu}_{IC}]$ and a pooling region $[\tilde{\mu}_{IC}, \mu_c]$. In the waiting region, high types' value function satisfies the following HJB:

$$\rho V^h(\mu) = \frac{dV^h(\mu)}{d\mu}\eta(1-\mu) + s\left[\theta^h\left(R - \frac{I}{\theta^h}\right) - V^h(\mu)\right]. \tag{13}$$

In the pooling region, high types' payoff is $\theta^h(R - F^p(\mu_t))$. The boundary, $\tilde{\mu}_{IC}$, is pinned down by the value-matching and smooth-pasting condition. The rest of the model is solved numerically. Figure 4 shows the value function of high-type borrowers.

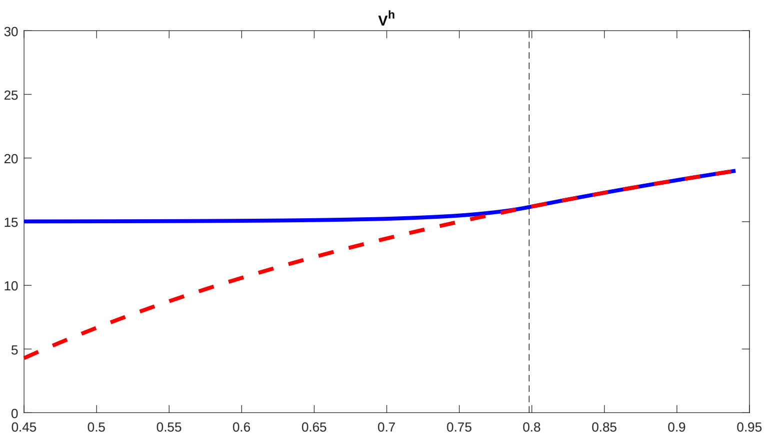

Figure 4: Value Functions in the Model with Success Probability

This figure shows high-quality borrowers' value function. The horizontal axes represent μ_t–average quality of the borrower pool. The blue curve depicts V_t^h. The red-dashed curve plots the continuation value from immediately accepting a pooling offer at face value $F^p(\mu_t)$. The black-dashed vertical line marks the position of $\tilde{\mu}_{IC}$, the cutoff value at which a high-quality borrower is indifferent between waiting and taking a pooling offer. The parameter values are $I = 20$, $R = 40$, $c = 3.8$, $\beta = 1$, $s = 1$, $\rho = 0.5$, $\eta = 0.5$, $\underline{q} = 0.1$, $\theta^h = 1$, and $\theta^l = 0.2$. In this case, $\mu_c = 0.94$ and $\tilde{\mu}_{IC} = 0.80$.

4.3 Exiting and Lending Relationships

By making a replacement assumption, this paper has hitherto focused on how entry changes the composition of the borrower pool. This subsection relaxes this replacement assumption in two ways. First, we drop the replacement so that once a borrower receives financing, she/her permanently *exits* the pool without a replacement. Second, we keep the replacement assumption but allow the replacement borrowers to form lending relationships with the previous banks. In this case, high-quality replacements essentially exit the borrower pool. Forming lending relationships is therefore similar to exiting, because borrowers in the pool are either those types unknown to all banks or known as low to some banks. In both cases, we show the main results continue to hold.

Exiting

One immediate observation is that N_t, the total measure of firms waiting to be financed, is 0 if pooling offers are immediately issued and accepted. If so, the total size of firm entry – proportional to N_t – is not well defined. Therefore, we restrict the maximum arrival rate of a pooling offer to be λ, so that even pooling offers cannot be immediately available.[14] Instead, they arrive at most at a rate λ. λ can be arbitrarily large, and our results hold under $\lambda \to \infty$, corresponding to a case in which pooling offers are (almost) immediately available. One can interpret a finite λ as frictions in credit search, constraints in lending due to insufficient bank capital, or the time-consuming process of interacting with banks.

Recall that n_t^h and n_t^l are the total measure of high- and low-type borrowers in the pool. Under this setup, they evolve according to the following process:

$$dn_t^h = \eta N_t dt - s n_t^h dt - \mathbb{1}_{F_t^p = F^p(\mu_t)} \lambda n_t^h dt \tag{14}$$

$$dn_t^l = \eta N_t \frac{1-q_t}{q_t} dt - \mathbb{1}_{F_t^p = F^p(\mu_t)} \lambda n_t^l dt, \tag{15}$$

where q_t is $\underline{q}$ if $V_t^l > c$, and 1 if vice versa. If $V_t^l = c$, q_t takes any value between $\underline{q}$ and 1. In (14), the first term captures the effect of entry, with the remaining two terms standing for exiting through screening and pooling, respectively. (15) can be interpreted similarly. In both equations, the indicator function $\mathbb{1}_{F_t^p = F^p(\mu_t)}$ follows the result in Lemma 1. If pooling offers are not issued because banks expect high types not to accept them, $F_t^p \to \infty$.

Equations (14) and (15) imply μ_t evolves according to the following process:

$$d\mu_t = \left[\eta\left(1 - \frac{\mu_t}{q_t}\right) - s\mu_t\left(1-\mu_t\right)\right] dt. \tag{16}$$

The term $-s\mu_t\left(1-\mu_t\right)$ is derived from $-s n_t^h dt$ in (14), which captures the standard cream-skimming effect: by lending only to high-type borrowers, a bank contaminates the rest of the borrowing pool. Note (16) is independent of λ and holds as $\lambda \to \infty$. To find a stationary equilib-

[14] To maintain the competitive nature of pooling offers, we assume pooling offers always arrive in pairs.

rium, it remains to solve $d\mu_t = 0$. Let q_{IC} and q_c be the corresponding q_t in (16) that lead to the solution being μ_{IC} and μ_c, respectively. The stationary equilibrium in which the average quality stays at μ_{IC} (μ_c) is constructed if the marginal quality is q_{IC} (q_c). Moreover, we can construct a convergence path that resembles the one in subsection 3.2. In both cases, the feedback between bank lending and potential entrepreneurs' entry still exists. Proposition 5 describes the results, under a parametric restriction that $\eta > s$. Intuitively, this condition requires the entry margin to exceed high types' exit margin.[15]

Proposition 5. *Assuming $\eta > s$, a unique stationary equilibrium exists, characterized by μ_{IC} and μ_c.*

1. *Strong fundamentals: if $\frac{s}{\rho+s}(R-I) > c$, $\mu^* = \mu_{IC}$. Both types of borrowers are financed by pooling offers $F^p(\mu_{IC})$ after a Poisson event that arrives at rate ϕ. High-type borrowers are also financed by screened offers at rate s. Among low-type potential entrepreneurs, $\eta \frac{1-q_{IC}}{q_{IC}} N_t dt$ of them enter during $[t, t+dt)$.*

2. *Weak fundamentals: if $\frac{s}{\rho+s}(R-I) < c$, $\mu^* = \mu_c$. Both types of borrowers are immediately financed by pooling offers $F^p(\mu_c)$. Among low-type potential entrepreneurs, $\eta \frac{1-q_c}{q_c} N_t dt$ of them enter during $[t, t+dt)$.*

3. *A convergence path to the weak-fundamental stationary equilibrium exists, characterized by a waiting region $[\mu_0, \min\{\tilde{\mu}_{IC}, \mu_c\})$ and a pooling region $[\min\{\tilde{\mu}_{IC}, \mu_c\}, \mu_c]$.*

Lending Relationships

We now turn to a discussion of how forming lending relationships may not change the results in the model with replacement. Given that borrower heterogeneity is modeled as differences in experiencing the liquidity shock, a borrower's type is fully revealed after one round of investment. If types are fully persistent; that is, the replacement has the identical quality, a high type's replacement will develop a lending relationship with the previous bank, thus exiting the borrower pool. In this case, dn_t^h is still described by (14). A low type's replacement, however, does not develop this lending relationship. If the private information on a borrower's type being low can be credibly shared with third-party agencies such as credit registries, dn_t^l is still described by (15). A model with lending relationships is then identical to one with exiting, so the results hold for any level of λ. When this private information cannot be credibly shared, a low type's replacement returns to

[15] If $\eta < s$, multiple stationary equilibria may exist.

the borrower pool, and dn_t^l follows:

$$dn_t^l = \eta N_t \frac{1-q_t}{q_t} dt.$$

Compared with (15), the missing term $-\mathbb{1}_{F_t^p = F^p(\mu_t)} \lambda n_t^l dt$ reflects another cream-skimming effect: low types' replacements return to the pool. In this case,

$$d\mu_t = \left[\eta \left(1 - \frac{\mu_t}{q_t} \right) - s\mu_t \left(1 - \mu_t \right) - \mathbb{1}_{F_t^p = F^p(\mu_t)} \lambda \mu_t \right] dt.$$

Obviously, the process of $d\mu_t$ is identical to (16) during the waiting region, where $F_t^p \neq F^p(\mu_t)$. However, the pooling region contains a new term $-\mathbb{1}_{F_t^p = F^p(\mu_t)} \lambda \mu_t$. If λ is not too high, we can construct a stationary equilibrium and convergence path similar to those in Proposition 5. Otherwise, the solution to $d\mu_t = 0$ in $(0,1)$ could fall below $\min \{\mu_{IC}, \mu_c\}$, and a stationary equilibrium would cease to exist. Intuitively, if the cream-skimming effect becomes too strong; that is, if λ gets too high, it dominates the effect from borrower entry, and a stationary equilibrium cannot exist.[16]

The broader lesson of this section is that both exiting through screening (at rate s) and through forming lending relationships (at rate λ) will reduce the average quality of the borrower pool. The equilibrium described in section 3 still exists as long as the entry margin dominates the exiting margin. Moreover, the equilibrium still features the two-way feedback between bank lending and firm entry.

Results are similar if the lending relationship breaks with some exogenous probability, in which case some high-type replacements are forced to return to the borrower pool. In this case, the requirement on λ can be further relaxed. Finally, note the analysis above depends on the assumption that borrowers' types are fully persistent. If, instead, a high-type borrower's replacement is only of high quality with probability $\chi < 1$, the incentives to develop lending relationships are lower. In fact, if $\chi < \tilde{\mu}_{IC}$, a high-type replacement no longer has the incentive to form a lending relationship.

4.4 Empirical Implications

The assumptions and predictions of our model are consistent with the stylized facts. Bank lending standards are counter-cyclical (Asea and Blomberg, 1998; Lown and Morgan, 2006; Rodano et al., 2015).[17] Figure 5 in Appendix A.3 replicates this fact, where bank lending standards are

[16] To construct a stationary equilibrium, the entry margin must also occur at a rate higher than λ. For example, when $\lambda \to \infty$ such that pooling offers can be immediately available, a stationary equilibrium requires high-type potential entrepreneurs to enter at the order of 1 instead of dt.

[17] Also see Becker et al. (2015), Greenwood and Hanson (2013), Kaplan and Stein (1993), Becker and Ivashina (2014).

measured by the percentage of loan officers who report a tightening in lending standards. Coming to the patterns of firm entry, previous studies have shown firm-entry quantity is pro-cyclical, whereas entry quality is counter-cyclical (Lee and Mukoyama, 2015; Moreira, 2015; Ates and Saffie, 2014). This finding is also replicated in Figure 6 in Appendix A.3. Our model reconciles these empirical patterns by studying equilibrium bank lending and firm entry in a unified framework. A model in which either bank lending or firm entry is exogenous (and therefore lacks the feedback between the two) is inconsistent with these facts. Moreover, studies show that credit quality deteriorates during an economic boom (Figueroa and Leukhina, 2015; Zhang, 2009), and changes in bank lending standards can predict economic outcomes (Bassett et al., 2014). Both are consistent with the model's predictions.

Our model also implies longer and larger economic booms are followed by slower recoveries. In the online appendix, we show this pattern holds across different countries, as well as across different industries in the US.[18] Therefore, recoveries in the model could be interpreted as either from industry-wide distresses or economy-wide recessions. In the most recent financial crisis, startups and young businesses – the counterparts of borrowers in the model– were hit harder than their larger counterparts and have been slower to recover (Mills and McCarthy, 2014). Because these firms account for 50% of gross job creation in the US (Decker et al., 2014; Haltiwanger et al., 2013), their weak performance directly contributes to the slow recovery. Moreover, a majority of these firms have been unable to secure any credit. Banks, which are the most important source of financing (Robb and Robinson, 2012), have been reluctant to extend loans.[19]

The main mechanism of our model is that high-type borrowers optimally choose the time to borrow from banks. Waiting and delay should be interpreted in terms of the time at which these borrowers choose to accept offers and make investments. After setting up the business venture (entry), borrowers approach the bank (without waiting) to learn the prevailing credit conditions. When the interest rates are high, they wait and postpone borrowing until either the bank has screened them or the borrower pool has improved substantially. Expecting so, banks will refuse to grant any credit without careful screening. This phenomenon is observationally equivalent to good borrowers being forced to wait because banks simply won't lend to anyone without careful screening, and screening can take some time to accomplish. Although plenty of research documents market timing in equity issuance, we are not aware of any empirical papers on market timing in borrowing bank loans. This prediction can be tested if better data become available.

[18] Also see Schularick and Taylor (2012) and Mendoza and Terrones (2012).

[19] Joint Small Business Credit Survey, 2014: https://www.newyorkfed.org/smallbusiness/joint-small-business-credit-survey-2014.html

5 Conclusion

How do banks change lending standards when they can dynamically screen borrowers and the pool of borrowers evolves endogenously? How do bank lending and firm entry affect each other? What are the implications for the investment fluctuations at either the sectoral or macroeconomic level? Why have some recoveries been slow and others fast, whereas some others have been accompanied by double dips?

This paper attempts to answer the above questions by constructing a model with borrowers who possess private information and banks that can dynamically produce this private information. In particular, the two-way feedback between firms' entry incentives and banks' financing terms highlights a channel through which credit standards and fluctuations in borrower quality can affect access to finance. Through this channel, convergence following bad economic shocks may or may not experience delays in investment and economic recoveries.

By carefully studying the composition of the borrower pool, this paper has focused on the asset side of banks. The implications on macroeconomic patterns therefore should be applied to recessions in which the financial sector is not heavily hit. Bigio and d'Avernas (2019) study recoveries from financial crises when banks are capital constrained. An interesting extension would be to introduce a role of bank capital and study how the level of bank capital dynamically interacts with the composition of assets over time.

References

Acharya, V. V. and S. Viswanathan (2011). Leverage, moral hazard, and liquidity. *The Journal of Finance 66*(1), 99–138.

Asea, P. K. and B. Blomberg (1998). Lending cycles. *Journal of Econometrics 83*(1), 89–128.

Ates, S. T. and F. E. Saffie (2014). Fewer but better: Sudden stops, firm entry, and financial selection.

Bassett, W. F., M. B. Chosak, J. C. Driscoll, and E. Zakrajšek (2014). Changes in bank lending standards and the macroeconomy. *Journal of Monetary Economics 62*, 23–40.

Becker, B., M. Bos, and K. Roszbach (2015). Bad times, good credit. *Swedish House of Finance Research Paper No. 15-05.*

Becker, B. and V. Ivashina (2014). Cyclicality of credit supply: Firm level evidence. *Journal of Monetary Economics 62*, 76–93.

Bigio, S. and A. d'Avernas (2019). Financial risk capacity. Technical report, National Bureau of Economic Research.

Bolton, P., T. Santos, and J. A. Scheinkman (2016). Cream-skimming in financial markets. *The Journal of Finance*.

Chang, B. (2018). Adverse selection and liquidity distortion. *The Review of Economic Studies 85*(1), 275–306.

Daley, B. and B. Green (2012). Waiting for news in the market for lemons. *Econometrica 80*(4), 1433–1504.

Daley, B. and B. Green (2016). An information-based theory of time-varying liquidity. *The Journal of Finance 71*(2), 809–870.

Daley, B., B. Green, and V. Vanasco (2020). Securitization, ratings, and credit supply. *The Journal of Finance 75*(2), 1037–1082.

Decker, R., J. Haltiwanger, R. Jarmin, and J. Miranda (2014). The role of entrepreneurship in us job creation and economic dynamism. *The Journal of Economic Perspectives 28*(3), 3–24.

Dell'Ariccia, G. and R. Marquez (2006). Lending booms and lending standards. *The Journal of Finance 61*(5), 2511–2546.

Diamond, D. W., Y. Hu, and R. G. Rajan (2020). Pledgeability, industry liquidity, and financing cycles. *The Journal of Finance 75*(1), 419–461.

Dixit, A. K., R. K. Dixit, and R. S. Pindyck (1994). *Investment under uncertainty*. Princeton university press.

Eisfeldt, A. L. (2004). Endogenous liquidity in asset markets. *The Journal of Finance 59*(1), 1–30.

Farboodi, M. and P. Kondor (2020). Rational sentiments and economic cycles. *NBER Working Paper* (w27472).

Fernald, J. (2012). A quarterly, utilization-adjusted series on total factor productivity. *Federal reserve bank of San Francisco working paper 19*, 2012.

Figueroa, N. and O. Leukhina (2015). Lending terms and aggregate productivity. *Journal of Economic Dynamics and Control 59*, 1–21.

Fishman, M. J. and J. A. Parker (2015). Valuation, adverse selection, and market collapses. *Review of Financial Studies 28*(9), 2575–2607.

Fishman, M. J., J. A. Parker, and L. Straub (2020). A dynamic theory of lending standards. *NBER Working Paper* (w27610).

Fuchs, W. and A. Skrzypacz (2015). Government interventions in a dynamic market with adverse selection. *Journal of Economic Theory 158*, 371–406.

Gorton, G. B. and G. Ordoñez (2014). Good booms, bad booms. Technical report, Working Paper.

Greenwood, R. and S. G. Hanson (2013). Issuer quality and corporate bond returns. *Review of Financial Studies 26*(6), 1483–1525.

Halac, M. and I. Kremer (2020). Experimenting with career concerns. *American Economic Journal: Microeconomics 12*(1), 260–88.

Haltiwanger, J., R. S. Jarmin, and J. Miranda (2013). Who creates jobs? small versus large versus young. *Review of Economics and Statistics 95*(2), 347–361.

Hu, Y. and F. Varas (forthcoming). A theory of zombie lending. *Journal of Finance*.

Kaniel, R. and D. Orlov (2020). Intermediated asymmetric information, compensation, and career prospects.

Kaplan, S. N. and J. C. Stein (1993). The evolution of buyout pricing and financial structure in the 1980s. *The Quarterly Journal of Economics 108*(2), 313–357.

Kaya, A. and K. Kim (2018). Trading dynamics with private buyer signals in the market for lemons. *The Review of Economic Studies 85*(4), 2318–2352.

Khanna, N., T. H. Noe, and R. Sonti (2008). Good ipos draw in bad: Inelastic banking capacity and hot markets. *The Review of Financial Studies 21*(5), 1873–1906.

Kremer, I. and A. Skrzypacz (2007). Dynamic signaling and market breakdown. *Journal of Economic Theory 133*(1), 58–82.

Kurlat, P. (2013). Lemons markets and the transmission of aggregate shocks. *American Economic Review 103*(4), 1463–89.

Lee, M. J. and D. Neuhann (2019). A dynamic theory of collateral quality and long-term interventions. *FRB of New York Staff Report* (894).

Lee, Y. and T. Mukoyama (2015). Entry and exit of manufacturing plants over the business cycle. *European Economic Review 77*, 20–27.

Lown, C. and D. P. Morgan (2006). The credit cycle and the business cycle: new findings using the loan officer opinion survey. *Journal of Money, Credit and Banking*, 1575–1597.

Malherbe, F. (2014). Self-fulfilling liquidity dry-ups. *The Journal of Finance 69*(2), 947–970.

Mendoza, E. G. and M. E. Terrones (2012). An anatomy of credit booms and their demise. Technical report, National Bureau of Economic Research.

Mills, K. and B. McCarthy (2014). The state of small business lending: Credit access during the recovery and how technology may change the game. *Harvard Business School General Management Unit Working Paper* (15-004).

Moreira, S. (2015). Firm dynamics, persistent effects of entry conditions, and business cycles.

Povel, P., R. Singh, and A. Winton (2007). Booms, busts, and fraud. *The Review of Financial Studies 20*(4), 1219–1254.

Rajan, R. G. (1992). Insiders and outsiders: The choice between informed and arm's-length debt. *The Journal of Finance 47*(4), 1367–1400.

Robb, A. M. and D. T. Robinson (2012). The capital structure decisions of new firms. *Review of Financial Studies*, hhs072.

Rodano, G., N. A. B. Serrano-Velarde, and E. Tarantino (2015). Lending standards over the credit cycle.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica: Journal of the Econometric Society*, 97–109.

Rubinstein, A. and A. Wolinsky (1985). Equilibrium in a market with sequential bargaining. *Econometrica: Journal of the Econometric Society*, 1133–1150.

Ruckes, M. (2004). Bank competition and credit standards. *Review of Financial Studies 17*(4), 1073–1102.

Schularick, M. and A. M. Taylor (2012). Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008. *American Economic Review 102*(2), 1029–61.

Tadelis, S. (1999). What's in a name? reputation as a tradeable asset. *American Economic Review 89*(3), 548–563.

Zhang, Z. (2009). Recovery rates and macroeconomic conditions: The role of loan covenants. In *AFA 2010 Atlanta Meetings Paper*.

Zryumov, P. (2015). Dynamic adverse selection time varying market conditions and endogenous entry.

A Appendix

A.1 Proofs

A.1.1 Proof of Lemma 1

Proof. Let p_t^θ, $\theta \in \{h, l\}$ be the probability of accepting a pooling offer at time t. Bayesian updating shows

$$\hat{\mu}_t = \frac{\mu_t p_t^h}{\mu_t p_t^h + (1 - \mu_t) p_t^l}.$$

Suppose $\hat{\mu}_t > \mu_t$, then $p_t^l < p_t^h \leq 1$. Mixed strategy by low types implies that there exists a $\tau^l > t$ where $V_t^l = e^{-\rho \tau^l} [R - F^p (\hat{\mu}_{\tau^l})]$. Let $\hat{\tau}^h \coloneqq \tau^l \wedge \tau_1^s$, where $\tau_1^s \in T_s$ is the first time of arrival However,

$$\begin{aligned} V_t^h &\geq \mathbb{E}\left[e^{-\rho\tau^l}\left(\mathbb{1}_{\hat{\tau}^h \notin T^s}\left(R - F^p\left(\hat{\mu}_{\tau^l}\right)\right) + \mathbb{1}_{\hat{\tau}^h \in T^s}\mathbb{1}_{\theta=h}\left(R - F^h_{\tau_1^s}\right)\right)\right] \\ &> \mathbb{E}\left[e^{-\rho\tau^l}\left[R - F^p\left(\hat{\mu}_{\tau^l}\right)\right]\right] = R - F^p\left(\hat{\mu}_t\right). \end{aligned}$$

This implies $p_t^h = 0$, a contradiction.

Suppose $\hat{\mu}_t < \mu_t$, then $p_t^h < p_t^l \leq 1$ and $V_t^h = R - F^p(\hat{\mu}_t)$. This immediately violates the No-Deals condition (3) as $F^p(\hat{\mu}_t) > F^p(\mu_t)$. □

A.1.2 Proof of Lemma 2

Proof. Given the Poisson arrival property, high types wait if and only if

$$\frac{s}{\rho + s}(R - I) > R - F^p(\mu^*) = R - [I + (1 - \mu^*)\ell].$$

μ_{IC} is defined as the solution to $\frac{s}{\rho+s}(R - I) = R - F^p(\mu^*)$. □

A.1.3 Proof of Proposition 1

Let us state the proposition under any $\underline{q}$.

Proposition. *There exists a stationary equilibrium, and it is unique unless* $\frac{s}{\rho+s}(R - I) = c$.

1. *Low entry cost: if* $\frac{s}{\rho+s}(R - I) > c$,
   (a) *If* $\underline{q} \geq \mu_{IC}$, $\mu^* = \underline{q}$. *All borrowers are immediately financed via a pooling offer* $F^p(\underline{q})$. *All low-type potential entrepreneurs enter the borrower pool.*

(b) *If* $\underline{q} < \mu_{IC}$, $\mu^* = \mu_{IC}$. *Both types borrowers are financed by pooling offers* $F^p(\mu_{IC})$ *after a Poisson event that arrives at rate* ϕ, *where* $\frac{\phi}{\rho+\phi}\frac{s}{\rho+s}(R-I) = c$. *High-type borrowers are also financed by screened offers* $F^h = I$ *at rate* s. *Among low-type potential entrepreneurs,* $\eta\frac{1-\mu_{IC}}{\mu_{IC}}N_t dt$ *of them enter during* $\left[t, t+dt\right)$.

2. *High entry cost: if* $\frac{s}{\rho+s}(R-I) < c$, $\mu^* = \mu_c$. *Both types of borrowers are immediately financed via a pooling offer* $F^p(\mu_c)$. *Among low-type potential entrepreneurs,* $\eta\frac{1-\mu_c}{\mu_c}N_t dt$ *of them enter during* $\left[t, t+dt\right)$.

Proof. Lemma 2 implies $\mu^* \geq \mu_{IC}$. To see this, note that if $\mu^* < \mu_{IC}$, Lemma 1 implies high types will never accept pooling offers. This implies $V_t^l = 0$, and $\mu^* = 1$. However, if $\mu^* = 1$, there is no adverse selection, and pooling offers are made at $F^p = 1$, contradicting $V_t^l = 0$. Stationary requires the marginal quality must equal $\mu^* \in (0,1)$. This implies $\mu^* \geq \underline{q}$.

If $\underline{q} > \mu_{IC}$, then $\mu^* \geq \underline{q} > \mu_{IC}$. Lemma 1 and 2 imply that in any stationary equilibrium, pooling offers are made at $F^p(\mu^*)$ and accepted by high types. If $V_t^l = R - F^p(\mu^*) \geq c$, (4) implies the marginal quality is $\underline{q}$ and therefore $\mu^* = \underline{q}$. If $V_t^l = R - F^p(\mu^*) < c$, then in any stationary equilibrium, $\mu^* \geq \mu_c > \underline{q}$, and $V_t^l = c$. If $\mu^* > \mu_c$, then pooling offers $F^p(\mu^*)$ must come with a delay; otherwise, $V_t^l > c$. In this case, banks make strictly positive profits by making an immediate pooling offer $F^p(\mu^*) + \epsilon$ for $\epsilon < F^p(\mu_c) - F^p(\mu^*)$. This offer will be accepted by high types, violating the No-Deals condition. Thus, in the stationary equilibrium, it must be $\mu^* = \mu_c$, and pooling offers are immediately made.

If $\underline{q} < \mu_{IC}$, then $\mu^* \geq \mu_{IC} > \underline{q}$. In any stationary equilibrium, low-type potential entrepreneurs must be indifferent between entering the pool and not. This implies $V_t^l = c$ and $\mu^* \geq \mu_c$. If $\frac{s}{\rho+s}(R-I) < c$, then $\mu^* \geq \mu_c > \mu_{IC} > \underline{q}$. A similar argument as the one above show $\mu^* > \mu_c$ cannot be the equilibrium due to the No-Deals condition. If $\frac{s}{\rho+s}(R-I) > c$, then $\mu^* \geq \mu_{IC} > \mu_c > \underline{q}$. A similar argument as the one above show $\mu^* > \mu_{IC}$ cannot be the equilibrium due to the No-Deals condition. Thus, $\mu^* = \mu_{IC}$. The stationary condition implies V_t^h and V_t^l are both a constant, so that pooling offers must follow a memory-less process, i.e., the Poisson process. The arrival rate ϕ is chosen such that $V_t^l = c$.

Finally, the case $\underline{q} = \mu_{IC}$ and $\frac{s}{\rho+s}(R-I) = c$ are both a knife-edge one and in general admits a continuum of stationary equilibrium.

□

A.1.4 Proof of Lemma 3

Proof. Lemma 1 implies along the convergence path, the face value of pooling offer, if exist, must be $F^p(\mu_t)$. With slight abuse of notation, let τ^l be one of the equilibrium stopping time chosen by low types in (3), and $\mu_{\tau^l} \leq \mu_c$. If $\mu_t < \mu_c$, then

$$V_t^l = \sup\ \mathbb{E}\left[e^{-\rho\tau^l}\left[R - F^p\left(\mu_{\tau^l}\right)\right]\right] \leq \mathbb{E}\left[e^{-\rho\tau^l}\left[R - F^p\left(\mu_c\right)\right]\right] < R - F^p\left(\mu_c\right) = c.$$

□

A.1.5 Proof of Lemma 4

Proof. The result that $\tilde{\mu}_{IC}$ is the unique solution to the value matching and smooth-pasting condition follows naturally by combining the two conditions.

The proof for smooth-pasting follows closely the proof of Proposition 6.1 in Daley and Green (2012). High types' value function of immediately accepting a pooling offer is $R - F^p\left(\mu_t\right) = R - I - \left(1 - \mu_t\right)\ell$, which linearly increases in μ_t.

Suppose $\frac{dV_t^h(\tilde{\mu}_{IC})}{d\mu_t} > \ell$. Then for ϵ sufficiently small, $V_t^h\left(\tilde{\mu}_{IC} - \epsilon\right) < R - F^p\left(\tilde{\mu}_{IC} - \epsilon\right)$, an immediate contradiction to No-Deals.

Suppose instead $\frac{dV_t^h(\tilde{\mu}_{IC})}{d\mu_t} < \ell$. In this case, high types receive a strictly higher payoff by continuing to wait until μ_t reaches $\tilde{\mu}_{IC} + \epsilon$, given $\frac{dV_t^h(\tilde{\mu}_{IC}+\epsilon)}{d\mu_t} < \ell$. This contradicts the policy of waiting until $\tilde{\mu}_{IC}$ is optimal. □

A.1.6 Proof of Proposition 2

Proof. We prove the case $\tilde{\mu}_{IC} < \mu_c$, and the other case follows naturally. Belief consistency on μ_t follows from the dynamics of μ_t, which satisfies $d\mu_t = \eta\left(1 - \mu_t\right)dt$ before μ_t reaches μ_c. Belief consistency on $\hat{\mu}_t$ implies that pooling offers $F^p\left(\mu_t\right)$ are only made after μ_t rises above $\tilde{\mu}_{IC}$.

Lemma 3 has verified the optimality of low-type potential entrepreneurs. Next, we verify the optimality of high-type borrowers. That is, we need to verify that waiting is optimal for $\mu_t < \tilde{\mu}_{IC}$, whereas accepting an immediate pooling offer is optimal for $\mu_t > \tilde{\mu}_{IC}$. Let us define

$$G\left(\mu\right) = V^h\left(\mu\right) - \left[R - F^p\left(\mu\right)\right] = V^h\left(\mu\right) - \left[R - I - \left(1 - \mu\right)\ell\right].$$

Clearly, $G\left(\tilde{\mu}_{IC}\right) = 0$ and $G' = \left(V^h\right)' - \ell$. Moreover, $G'\big|_{G=0} = \frac{\rho(R-I)-(\rho+s+\eta)(1-\mu)\ell}{\eta(1-\mu)} < 0$ if and only if $\mu < \tilde{\mu}_{IC}$. □

A.1.7 Proof of Proposition 3

Proof. Let V^h be high types' payoff in the stationary equilibrium. The stationary condition requires pooling offers arrive at some Poisson rate ϕ^β. The case of $\phi^\beta \to \infty$ corresponds to an immediate pooling offer.

$$V^h = \frac{\phi^\beta}{\rho+\phi^\beta+s}\left[R-F^p\left(\mu^*\right)\right] + \frac{s}{\rho+\phi^\beta+s}\left[R-(1-\beta)\left(R-V^h\right)-\beta I\right]$$
$$\Rightarrow V^h = \frac{\phi^\beta}{\rho+\phi^\beta+s\beta}\left[R-F^p\left(\mu^*\right)\right] + \frac{s\beta}{\rho+\phi^\beta+s\beta}(R-I).$$

In the stationary equilibrium, high types are indifferent between a pooling offer $F^p(\mu^*)$ and waiting iff

$$R-F^p\left(\mu^*\right) = \frac{s}{\rho+s}\left[R-(1-\beta)\left(R-V^h\right)-\beta I\right]. \tag{17}$$

Define $\mu_{IC}^\beta = 1-\frac{\rho}{\rho+s\beta}\frac{R-I}{\ell}$ as the solution to the equation above as $\phi^\beta\to\infty$, i.e., the threshold if a pooling offer is immediately available. Low types' payoff from accepting an immediate pooling offer is $\frac{s\beta}{\rho+s\beta}(R-I)$. If $\frac{s\beta}{\rho+s\beta}(R-I)<c$, this corresponds to the high entry cost case, and the unique equilibrium has $\mu^*=\mu_c$, similar to Proposition 1. If $\frac{s\beta}{\rho+s\beta}(R-I)>c$, this corresponds to the high entry cost case, and the equilibrium therefore must involve delay in the sense that pooling offers arrive at some Poisson rate ϕ^β. Note that in this case, Equation (17) also depends on ϕ^β and μ^* increases with ϕ^β. Finally, the entry condition implies

$$V_t^l = \frac{\phi^\beta}{\rho+\phi^\beta}\left[R-F^p\left(\mu^*\right)\right]=c. \tag{18}$$

Any solution to (17) and (18) consist a stationary equilibrium. After some derivation, the two equations simplify to

$$\frac{\phi^\beta}{\rho+\phi^\beta}\frac{\rho+\phi^\beta+s}{\rho+\phi^\beta+s\beta}\frac{\phi^\beta s\beta}{(\rho+s)^2+\phi^\beta(\rho+s\beta)}(R-I)=c.$$

For $\frac{s\beta}{\rho+s\beta}(R-I)>c$, the existence of the solution is guaranteed because the left-hand-side is 0 as $\phi^\beta=0$ and converges to $\frac{s\beta}{\rho+s\beta}(R-I)$ as $\phi^\beta\to\infty$.

Finally, for the case of $\frac{s\beta}{\rho+s\beta}(R-I)<c$ and $\mu_0\in\left(\mu_{\min}^p,\mu_c\right)$, the HJB in the convergence region satisfies

$$\rho V^h(\mu) = \frac{dV^h(\mu)}{d\mu}\eta(1-\mu)+s\beta\left[(R-I)-V^h(\mu)\right].$$

The same proof shows Lemma 3 and 4 continue to hold, which pins down the solution for $\tilde{\mu}_{IC}^{\beta}$:

$$\tilde{\mu}_{IC}^{\beta} = 1 - \frac{\rho}{\rho + s\beta + \eta} \frac{R - I}{\ell}.$$

$\square$

A.1.8 Proof of Proposition 4

Proof. In the region where high types wait, their HJB satisfies

$$\rho V^h(\mu) = \frac{dV^h(\mu)}{d\mu} \eta (1 - \mu) \tag{19}$$

if the banks do not screen $s_t = 0$. The solution to this ODE is

$$V_t^{h,0} = \frac{\eta}{\rho + \eta} \left[\frac{\rho (R - I)}{(1 - \mu_t) \ell (\rho + \eta)} \right]^{\frac{\rho}{\eta}}. \tag{20}$$

The HJB becomes

$$\rho V^h(\mu) = \frac{dV^h(\mu)}{d\mu} \eta (1 - \mu) + s\beta \left[(R - I) - V^h(\mu) \right] \tag{21}$$

if banks screen $s_t = s$. The solution to this ODE is

$$V^{h,s}(\mu) = \frac{s\beta}{\rho + s\beta}(R - I) + \left[\frac{\rho (R - I)}{(1 - \mu) \ell (\rho + \eta + s\beta)} \right]^{\frac{\rho + s\beta}{\eta}} \left[\frac{s\beta}{\rho + s\beta}(R - I) + \frac{s\beta + \eta}{\rho + s\beta + \eta} \right]. \tag{22}$$

Let us first prove the following lemma which will be very useful to characterize the bank's optimal strategy.

Lemma 5. *Under the constructed equilibrium in Proposition 4, bank's profit function $\Pi(\mu_t)$ increase on $\mu_t \in \left[\mu_0, \underset{\sim}{\mu}_{IC}\right]$ and $\mu_t \in \left[\mu_{dd}, \tilde{\mu}_{IC}^{\beta}\right]$.*

Proof. A sufficient condition is to show that

$$\frac{d\mu_t \left[(R - I) - V_t^h \right]}{d\mu_t} > 0,$$

for $\mu_t < \underset{\sim}{\mu}_{IC}$ and $\mu_t \in [\mu_{dd}, \mu_s]$. Simple calculation shows this is equivalent to verify

$$
\begin{aligned}
R - I - V_t^h &> \mu_t \left(V_t^h\right)' = \mu_t \frac{\rho V_t^h}{\eta (1-\mu_t)} \\
R - I &> V_t^h \frac{\eta + (\rho - \eta)\mu_t}{\eta + (1-\mu_t)}.
\end{aligned}
$$

Note that $V_t^h \leq V_t^h \left(\underset{\sim}{\mu}_{IC}\right) = \frac{\eta}{\rho+\eta}(R-I)$. A sufficient condition is therefore

$$
\begin{aligned}
R - I &> \frac{\eta}{\rho+\eta}(R-I) \frac{\eta + (\rho-\eta)\mu_t}{\eta + (1-\mu_t)} \\
1 &> \frac{\eta}{\rho+\eta} \frac{\eta + (\rho-\eta)\mu_t}{\eta + (1-\mu_t)} \\
1 &> \frac{\eta}{\rho+\eta} \frac{\eta + (\rho-\eta)\underset{\sim}{\mu}_{IC}}{\eta + (1-\mu_t)}.
\end{aligned}
$$

The last condition always holds as long as $\underset{\sim}{\mu}_{IC} < 1$.

On $\mu_t \in \left[\mu_s, \tilde{\mu}_{IC}^{\beta}\right]$, the same condition $\frac{d\mu_t\left[(R-I)-V_t^h\right]}{d\mu_t} > 0$ can be simplified into

$$
1 > \frac{\eta + (\rho-\eta)\mu}{\eta + (1-\mu)} \frac{s\beta + \eta}{\rho + s\beta + \eta},
$$

which always holds under the condition $\eta^2 > s(\rho - \eta)$. □

High types' optimal strategies on $\left[\mu_0, \underset{\sim}{\mu}_{IC}\right]$ and $\left[\mu_s, \tilde{\mu}_{IC}^{\beta}\right]$ are easily verified following the proof in Proposition 2 with $s_t \equiv 0$ and $s_t \equiv s$ respectively. It remains to verify the optimality for $\left[\underset{\sim}{\mu}_{IC}, \mu_s\right]$ and the banks' decision of screening.

Let us impose the following condition

$$
\kappa \in \left(\underset{\sim}{\mu}_{IC}(1-\beta)(R-I)\frac{\rho}{\rho+\eta}, \tilde{\mu}_{IC}^{\beta}(1-\beta)(R-I)\frac{\rho}{\rho+s+\eta}\right). \tag{23}
$$

$\kappa > \underset{\sim}{\mu}_{IC}(1-\beta)(R-I)\frac{\rho}{\rho+\eta}$ guarantees that at $\mu_t = \underset{\sim}{\mu}_{IC}$, $\mu_t(1-\beta)\left[(R-I)-V_t^h\right] < \kappa$ so that $s_t = 0$. $\kappa < \tilde{\mu}_{IC}^{\beta}(1-\beta)(R-I)\frac{\rho}{\rho+s+\eta}$ guarantees that at $\mu_t = \tilde{\mu}_{IC}^{\beta}$, $\mu_t(1-\beta)\left[(R-I)-V_t^h\right] > \kappa$ so that banks $s_t = s$. Note that this set is non-empty because one can easily verify

$$
\underset{\sim}{\mu}_{IC}\frac{\rho}{\rho+\eta} < \tilde{\mu}_{IC}^{\beta}\frac{\rho}{\rho+s+\eta}.
$$

Lemma 5 and Condition (23) imply that the bank never screens for $\mu_t < \underset{\sim}{\mu}_{IC}$. Moreover, there exists a μ_s such that $\Pi(\mu_s) = \kappa$. The bank screens on $\mu_t \in \left[\mu_s, \tilde{\mu}_{IC}^{\beta}\right]$. This verifies that it is optimal for the bank not to screen on $\mu_t \in [\mu_{dd}, \mu_s]$.

Finally, to verify high-type borrowers' strategy on $\left[\underset{\sim}{\mu}_{IC}, \tilde{\mu}_{IC}^{\beta}\right]$, note that the solution (20) and smooth-pasting imply $\left(V_t^h\right)' > \ell$ on $\mu_t > \underset{\sim}{\mu}_{IC}$. Also, the solution (22) and smooth-pasting imply $\left(V_t^h\right)' < \ell$ on $\mu_t < \tilde{\mu}_{IC}^{\beta}$. Therefore, there exists a unique $\mu_\dagger \in \left[\underset{\sim}{\mu}_{IC}, \tilde{\mu}_{IC}^{\beta}\right]$ where the two solutions interact. The monotonicity in bank profits imply that if

$$\mu_\dagger (1-\beta)\left[(R-I) - V_t^{h,s}(\mu_\dagger)\right] < \kappa.$$

Otherwise, banks choose to screen $s_t = s$ at $\mu_t = \mu_\dagger$, so that at $\mu_t = \underset{\sim}{\mu}_{IC}$, high type borrowers obtain a higher continuation value by waiting. In this case, region 2 disappears and the equilibrium is characterized by first dip $\mu_t \in [\mu_0, \mu_s]$, second dip $\mu_t \in \left[\mu_s, \tilde{\mu}_{IC}^{\beta}\right]$ and second rise $\mu_t \in \left[\tilde{\mu}_{IC}^{\beta}, \mu_c\right]$.

To summarize, the cost of screening needs to satisfy

$$\kappa \in \left(\mu_\dagger (1-\beta)\left[(R-I) - V_t^{h,s}(\mu_\dagger)\right], \tilde{\mu}_{IC}^{\beta}(1-\beta)(R-I)\frac{\rho}{\rho+s+\eta}\right), \tag{24}$$

where $\mu_\dagger$ is the unique solution to

$$V_t^{h,s} = V_t^{h,0}.$$

□

A.1.9 Proof of Proposition 5

Proof. From $d\mu_t = \left[\eta\left(1 - \frac{\mu_t}{q_t}\right) - s\mu_t(1-\mu_t)\right]dt = 0$, we get a quadratic equation

$$s\mu_t^2 - \left(\frac{\eta}{q_t} + s\right)\mu_t + \eta = 0.$$

Let $g(\mu_t) = s\mu_t^2 - \left(\frac{\eta}{\underline{q}} + s\right)\mu_t + \eta$. Clearly, $g(0) = \eta > 0$, and $g(q_t) = sq_t(q_t - 1) < 0$. Therefore, for any q_t, there exists a unique $\mu^* \in (0, q_t)$ such that $d\mu_t = 0$.

Lemma 2 and Equation (6) continue to capture the threshold in high types accepting an immediate pooling offer and low types entering. To prove the stationary equilibrium, all we need to show is for both the case $\mu^* = \mu_{IC}$ and $\mu^* = \mu_c$, there exists a q_t such that $g(\mu^*) = 0$. Given the continuity of $g(\mu_t)$ and $\eta > s$, this is guaranteed because $\mu^* < \underline{q}$ if $q_t = \underline{q}$ and $\mu^* = 1$ if $q_t = 1$.

Finally, to prove the convergence path, let us write down the HJB in the waiting region

$$\rho V^h(\mu) = \frac{dV^h(\mu)}{d\mu}\left[\eta(1-\mu) - s\mu(1-\mu)\right] + s\beta\left[(R-I) - V^h(\mu)\right],$$

where we have used the result from Lemma 3 so that $q_t = 1$ before μ_t reaches μ_c. Under value-matching and smooth-pasting condition, we can derive the following equation for $\tilde{\mu}_{IC}$ to satisfy

$$\rho(R-I) = \ell(1-\mu_t)\left[\rho + \eta + s(1-\mu_t)\right].$$

Let $g(\mu_t) = \rho(R-I) - \ell(1-\mu_t)\left[\rho + \eta + s(1-\mu_t)\right]$. Clearly, $g(1) = \rho(R-I) > 0$ and $g(0) = \rho(R-I-\ell) - \ell(\eta+s) < 0$. So $\tilde{\mu}_{IC}$ exists and is unique. The step of verification follows from the proof of Proposition 2. □

A.2 Extended Analysis

A.2.1 A Model with A Fixed Size of Potential Entry

Let us still assume that during a short period $[t, t+dt)$, a flow of potential entrepreneurs are born. Among them, ηdt are of high quality, and $\frac{1-\underline{q}}{\underline{q}}\eta dt$ are of low quality. In this model, Lemma 1 and 2 continue to hold, so that the stationary equilibrium stays unchanged as in Proposition 1. In the strong fundamentals case, the stationary equilibrium is also a steady-state equilibrium with $n_t^h = n_{ss}^h$ and $n_t^l = n_{ss}^l$, where

$$\begin{aligned}(\phi+\lambda)\, n_{ss}^h &= \eta \\ \Rightarrow n_{ss}^h &= \frac{\eta}{\phi+\lambda}.\end{aligned}$$

This implies

$$n_{ss}^l = \frac{1-\mu_{IC}}{\mu_{IC}} n_{ss}^h.$$

Between $[t, t+dt)$, the total measure of low-type potential entrepreneurs that choose to enter is exactly $\phi n_{ss}^l dt = \frac{1-\mu_{IC}}{\mu_{IC}}\frac{\eta}{\phi+\lambda}dt$. The other equilibrium properties are unchanged. In the weak fundamental case, the total measure of low-type potential entrepreneurs that choose to enter is $\eta\frac{1-\mu_c}{\mu_c}dt$.

Along the transitional path studied in subsection 3.2, Lemma 3 continues to hold so that before

μ_t reaches μ_c,

$$dn_t^h = \eta dt$$
$$dn_t^l = 0.$$

This implies the state variable μ_t evolves according to

$$d\mu_t = (1 - \mu_t) \frac{\eta}{N_t} dt.$$

To fully characterize the dynamics along the transitional path, one needs to solve a differential equation with two variables μ_t and N_t, which becomes unnecessarily challenging. However, if μ_t is sufficiently low, a waiting region will continue to exist.

A.2.2 Convergence to a strong-fundamental Stationary Equilibrium

Let us construct a convergence path from $\mu_0 \in \left(\mu_{\min}^p, \mu_{IC}\right)$ when the stationary equilibrium is characterized by Case 1 of Proposition 1.

In particular, we construct a path that resembles Proposition 2, as follows. Before μ_t reaches μ_{IC}, pooling offers are never issued. Banks won't issue them $(F^p(\mu_t))$ because they expect only low types to accept. Therefore, $V_t^l(\mu_t) < c$ if $\mu_t < \mu_{IC}$. This implies only high-type potential entrepreneurs enter the pool before μ_t reaches μ_{IC}, and $d\mu_t = \eta(1 - \mu_t) dt > 0$. The HJB during the waiting region is characterized by (8), and the boundary condition is $V^h(\mu_c) = \frac{\phi+s}{\rho+\phi+s}(R - I)$. When μ_t finally reaches μ_{IC}, pooling offers are made at rate ϕ, where ϕ is determined in Proposition 1. The verification follows from the same Proof of Proposition 2.

A.3 Additional Figures

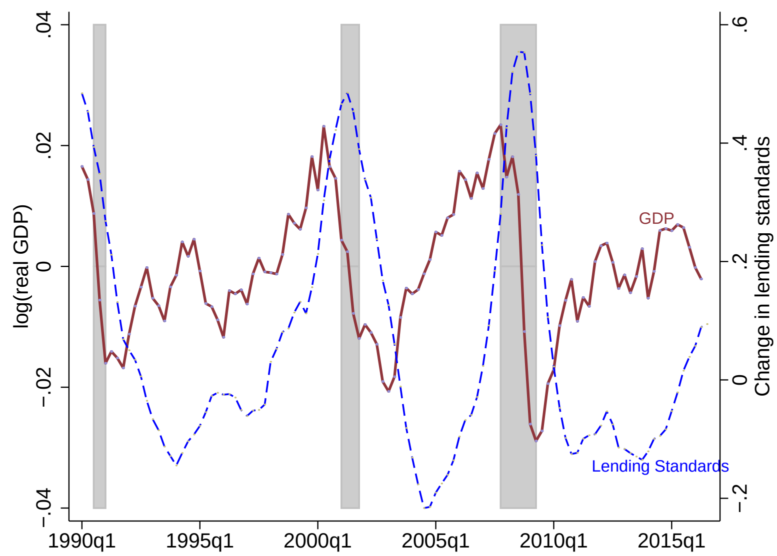

Figure 5: Lending Standards and Output

This figure plots the series of bank lending standards and output. The dashed curve depicts the sequence of bank lending standards, measured by the net percentage of loan officers who report a tightening in lending standards. The data are collected from Senior Loan Officer Opinion Survey on Bank Lending Practices (SLOOS). The solid curve shows output from Fernald (2012), measured as the percentage change at an annual rate (=400×changes in natural log). Both sequences are further smoothed with a moving-average filter with two lagged terms, three forward terms, and including the current observation in the filter. Shaded areas are recessions identified by the NBER.

Figure 6: Firm Entry Rate and Output

This figure plots the series of firm-entry rate and output. The dotted green curve depicts the sequence of firm-entry rate, measured by the number of new firms over exiting firms. The data are collected from the Firm Characteristics Data Tables of the Business Dynamics Statistics (BDS). The solid curve shows output from Fernald (2012), measured as percentage change at an annual rate (=400×changes in natural log). The output sequence is further smoothed with a moving-average filter with two lagged terms, three forward terms, and including the current observation in the filter. Shaded areas are recessions identified by the NBER.